Computational Research Progress in Applied Science & Engineering

ISSN 2423-4591

CRPASE Vol. 03(01), 25-34, February 2017

Modeling and Evaluating of Wave Run-up and Overtopping using Smoothed Particle Hydrodynamics Method

Mohamad Reza Naeej[a], Mojtaba Naeej[b*], Jafar Salehi[c], Reyhaneh Rahimi[d]

[a] *Deptartment of Civil Engineering, Shahrood University of Technology, Shahrood, Iran*
[b] *Deptartment of Civil Engineering, Babol University of Technology, Babol, Iran*
[c] *Deptartment of Civil Engineering, Islamic Azad University Chalus branch, Chalus, Iran*
[d] *Deptartment of Geology, Shahrood University of Technology, Shahrood, Iran*

Keywords

Wave overtopping,
Wave run-up,
SPH method,
Navier-Stokes equation,
Numerical method.

Abstract

This paper presents a smoothed particle hydrodynamics (SPH) method to investigate wave overtopping over coastal structures. The SPH method is a mesh-free particle modeling approach that can efficiently treat the large deformation of free surface. To simulate free surface flows in Lagrangian Navier-Stokes equation, fluid particles are followed using numerical methods in several time-steps. The proposed numerical method expected to provide a promising practical tool to investigate the complicated wave-structure interactions. The water depth over the horizontal section of the model is varied between 0.105–0.26 m. The wave height of the model is varied between 0.022–0.155 m. Firstly, a solitary-wave run-up is verified by comparing results from the numerical model to the experimental data. Then, a solitary-wave overtopping was modeled and the results were compared with experimental data. Correlation coefficient R^2 obtained from experimental and SPH model are 0.96 and 0.958 respectively. These results show that the present model is suitable for simulating complex problems in fluid mechanics with boundary conditions of free surface showing very good agreement with the experimental data.

1. Introduction

Solitary waves are wave forms that consist of a single wave, rather than waves that form part of a series of continuous regular waves or random waves. Solitary-type waves occur over a range of geophysical scales, with the most well-known theoretical application being for tsunami waves generated by submarine seabed displacement or impulsive waves generated by landslides or asteroid impact [1]. If the solitary wave has sufficient magnitude it may run-up and overtop natural beach dunes and coastal defenses such as dikes, breakwaters and seawalls, with potentially catastrophic effects for coastal infrastructure and populations [2].

The wave overtopping is a violent natural phenomenon which may cause the failure of coastal structures and the damage to the properties and lives. The wave overtopping waves can break, often subjected to the large deformation of free-surfaces. The real situations are highly complex, involving the complicated physical settings, the turbulence and eddy vortices, and the strong interactions between the wave and structure. Once the highest run-up levels exceed the free board wave overtopping occurs and the associated instantaneous discharge over the structures may pose a hazard to the coastal defenses. The European Wave overtopping Manual [3] provides a very comprehensive and practical tool for estimating the wave overtopping for different coastal defenses and has been widely used in the engineering field with significant accuracy. Recently great progresses have been made in the studies of wave overtopping through using the analytical, experimental and numerical approaches. For example, Umeyama [4] used both the theoretical and the experimental analyses to investigate the wave overtopping on a vertical boundary. Hedges and Reis used the random wave data and dimensional analysis to obtain several practical relationships to evaluate the wave overtopping volume [5]. Colagrossi et al. even presented a 2D+t SPH model to study the breaking wave pattern generated by fast-moving ships [6]. Cox and his co-associates carried out two experimental studies on the wave overtopping over a fixed deck using the transient and irregular waves [7, 8]. Walkden et al. made a study into the breakwater safety by measuring and analyzing the wave overtopping wave pressures and forces [9]. The numerical

* Corresponding Author:
E-mail address: m.naeej@stu.nit.ac.ir – Tel, (+98) 9113960156

Received: 08 August 2016; Accepted: 10 January 2017

method has been shown to be robust, applicable and comprehensive to a wide variety of problems. It has been used in Astrophysical [10], solid simulation [11, 12] and hydro-dynamical problems as the study of gravity currents [13, 14], free surface flows, especially wave propagation [14-16]. Recently, some investigators have examined the use of Smoothed Particle Hydrodynamics, SPH, for wave impact studies on offshore structures [17]. The SPH method is suitable to examine the wave overtopping of a coastal structures and defenses. This method is developed during seventies to solve astrophysical problems in three dimensional open spaces, avoiding the limitations of finite difference methods [10, 18]. In the SPH method, the state of a system is represented by a set of particles, which possess individual material properties and move according to the governing conservation equations. This method is a mesh-free, Lagrangian particle method for modeling fluid flows. The advantages of SPH arise directly from its Lagrangian nature, since a Lagrangian method can tackle difficulties related with lack of symmetry or a multiply-connected fluid much more efficiently than Eulerian methods can. There are no constraints imposed either on the geometry of the system or in how far it may evolve from the initial conditions. Since there is no mesh to distort, the approach can handle large deformations in a pure Lagrangian frame and material interfaces are followed naturally. The power of the method lies in its conceptual simplicity which gives rise to such desirable features as robustness, ease of simulating three-dimensional problems, a natural treatment of void regions, and ease of adding new physics [19]. Besides, numerical modeling based on the Navier-Stokes (N-S) equations has the advantage of including the irregular seabed geometries, nonlinear friction forces, nonlinear waves, and inhomogeneous porous media. They are capable of calculating the flows inside the complex geometries to disclose very refined information about the pressure, turbulence, transport property velocity, and so forth. The numerical models based on the 2D N-S type equations and the Reynolds averaged N-S (RANS) equations are possibly the most common to the investigation of wave overtopping and wave-structure interactions for engineering purposes, as the computational works are reasonably small, and the number of simplifying assumptions is considerably decreased as compared to other existing models. The numerical studies on wave overtopping using a similar RANS approach were also presented in [20-22], in which different forms of the two-equation k–ε model were employed to show the turbulence effects. Raichlen carried out a detailed study on the wave overtopping of a sloping sea wall by solving the primitive N–S equations with Large Eddy Simulation (LES) as the turbulence model [23]. Monaghan and Rafiee also proposed an SPH algorithm for multi-fluid flow with high density ratios [24]. In their simulations of multiphase flows with complex interfaces (i.e. Rayleigh–Taylor instability and gravity currents in their paper), the particles on interfaces are artificially treated as rigid boundary particles, and only single degree of freedom is given to the movement of these particles. This treatment helps to stabilize the interfaces, but is unphysical because the particles on interfaces are able to move in more than one direction in reality. Furthermore, many other investigators proved the high accuracy of SPH method in their works. The major objective of this study was to use numerical method of SPH to model the wave run-up and wave overtopping. In this study first the solitary wave run-up was modeled and verified by experimental results. The results obtained from this study were in good agreement with available experimental results. After modeling the wave run-up, the solitary wave overtopping was modeled and compared with experimental results showing good agreement. Finally, a proposed numerical method is developed which expected to provide a promising practical tool to investigate the complicated wave-structure interactions.

2. SPH Method

Smoothed Particle Hydrodynamics (SPH) is a relatively new method for examining the propagation of highly nonlinear and breaking waves. This method is a mesh-free, stable and Lagrangian solver for free surface hydrodynamics problems. The detailed formulation and these features of SPH will be addressed in this and following chapters and they will be demonstrated in some working examples in the later chapters. All the concepts, strategies and essential formulations discussed in this chapter are very useful in the development of the SPHysics code.

2.1. Integral Interpellants

SPH is based on integral interpellants. The fundamental principle is to approximate any function A(r) by (kernel approximation) as Eq. (1)

$$A(r) = \int_{\Omega} A(r')W(r - r', h)dr' \quad (1)$$

where *r* is the vector position; *W* is the weighting function or kernel; h is called smoothing length and it controls the influence domain (see Figure 1). Typically, value of h must be higher than initial particle separation. The approximation 1, in discrete notation, leads to the following approximation of the function at a particle a, (particle approximation) as

$$A(r) = \sum_b m_b \frac{A_b}{\rho_b} W_{ab} \quad (2)$$

where the summation is over all the particles within the region of compact support of the kernel function. The mass and density are denoted by m_b and ρ_b respectively and $W_{ab} = W(r_a - r_b, h)$ is the weight function or kernel.

$$\nabla A(r) = \sum_b m_b \frac{A_b}{\rho_b} \nabla W_{ab} \quad (3)$$

2.2. The Smoothing Kernel

The performance of an SPH model depends on the choice of the weighting functions. They should satisfy several conditions such as positivity, compact support, and normalization. Also, W_{ab} must be monotonically decreasing with increasing distance from particle a and behave like a delta function as the smoothing length, h, tends to zero

Positivity: $W(r - r', h) \geq 0$ inside the domain Ω (4a)

Compact support: $W(r-r',h)=0$ out of the domain Ω (4b)

Normalization: $\int_{\Omega} W(r-r',h)=1$ (4c)

Delta function behavior:

$$\lim_{h\to 0} W(r-r',h)dr' = \delta(r-r') \quad (4d)$$

Monotonically decreasing behavior of $W(r-r',h)$ (4e)

Kernels depend on the smoothing length, h, and the non-dimensional distance between particles given by $q=r/h$, r being the distance between particles a and b. The parameter h controls the size of the area around particle a where contribution from the rest of the particles cannot be neglected.

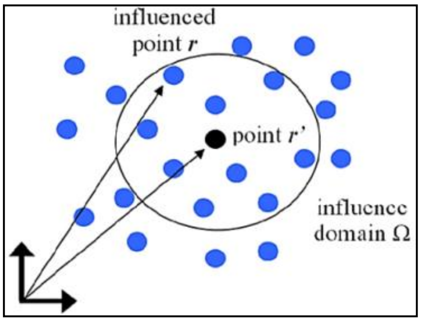


Figure 1. Sketch of the influence domain

3. Sphysics Codes

Sphysics is a platform of Smoothed Particle Hydrodynamics (SPH) codes inspired by the formulation of [25] developed jointly by researchers at the Johns Hopkins University (U.S.A.), the University of Vigo (Spain), the University of Manchester (U.K.) and the University of Rome La Sapienza (Italy). Some numerical methods are used to improve the numerical accuracy and to reduce the computational time in Sphysics code. Also different choices of initial and boundary conditions are described in detail in these sections. Two versions of Sphysics are available including Sphysics-2D and 3D. In Sphysics-2D the computational domain is considered to be 2D, where x corresponds to the horizontal direction and Z to the vertical direction. Besides, in SPHysics-3D the computational domain is fully 3D where, x and y are the horizontal directions and Z the vertical direction. In this study Sphysics-2D code was employed to model the wave run-up and wave overtopping.

4. Wave Run-up and Overtopping

An extensive summary of the literature on solitary wave propagation, run-up and impression in the context of tsunami impact is given by Nayfeh [1]. This work investigates the wave overtopping flow, which has not been extensively studied. Besides, most previous work has strategies and essential formulations discussed in this chapter are very useful in the development of the Sphysics code. Additionally, tsunami waves, or the leading positive waves in a tsunami wave train, may also make landfall in the form of broken waves, which impact coastal defenses and beaches, and lead to the initial over-wash or wave overtopping of coastal dunes dikes breakwaters and seawalls. Eventually, the large mass of water in the main tsunami wave overtakes the initial run-up, mostly leading to further inundation. However, during the initial first few minutes, the impact of the tsunami may be dominated by the run-up from broken waves. This initial period is important in the context of human safety on the immediate foreshore and in terms of warning systems. It is also relevant to the potential impact forces on coastal defenses, particularly if the run-up picks up debris along the coastline [26].

5. Wave Run-up Modeling

The wave run-up model is shown in Figure 2. The bathymetry comprised a 3 m long horizontal section from the wave maker to the toe of a uniform long sloping beach of gradient $\gamma = 0.107$ (Figure 2). According to [26], the sloping beach was constructed in two parts: a fixed lower section below the still water line (SWL), which is the position of the initial shoreline, and an adjustable beach with removable panels above the SWL [27]. The origin of the horizontal coordinate is at the SWL and positive onshore. The surface of the beach was a smooth painted marine plywood bed. Joints between adjacent panels were sanded flush to minimize additional roughness. In wave run-up modeling by SPH method the panel never changed but considered long enough to model the actual models well. The water depth over the horizontal section of the model was also varied between d =0.105–0.26 m; this additionally changed the beach truncation position relative to the SWL. The wave height of the model, H, varied between 0.022–0.155 m.

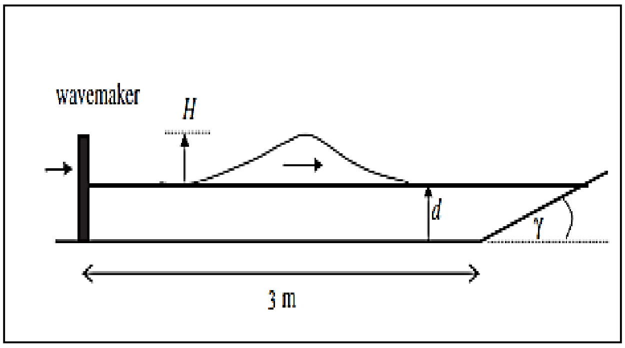
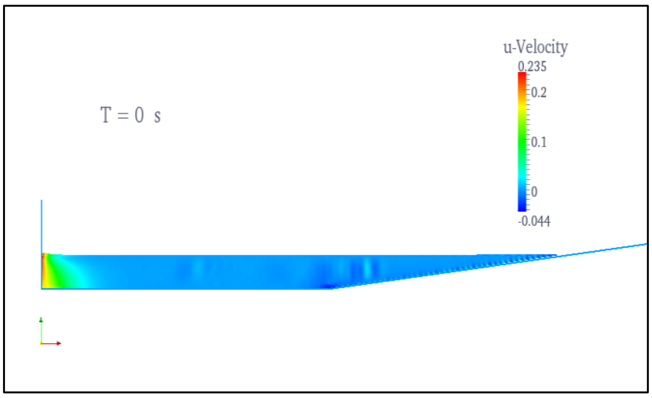
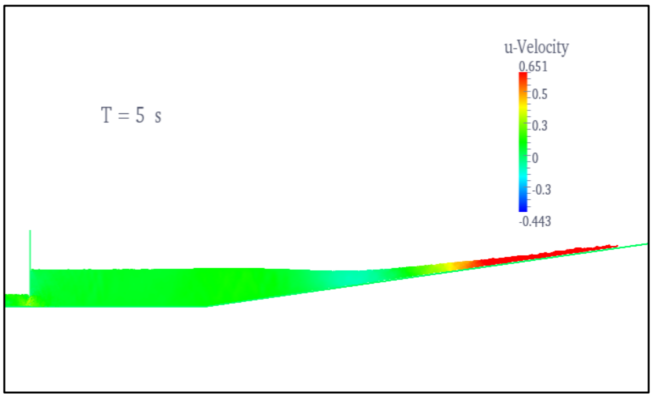


Figure 2. The initial geometry of solitary wave run-up.

5.1. Run-up Results using SPH Method

Actually, 21 cases were considered to model the wave run-up using SPH method. For summary, only the case number 6 is shown in Figure 3. As it shown, high wave run-up velocities occurs at the front of the wave run-up crest. This issue is quite reasonable. Table 1 lists the experimental and SPH model parameters and values for all the 21 cases. The experimental values were exported from [26] (Figure 4). The parameters d, H, and R represent water depth, wave height, and natural run-up elevation, respectively. Comparison between experimental data of [26] and SPH models are shown in Figure 4. It can be seen that SPH model diagram is in a good agreement with that of [26].





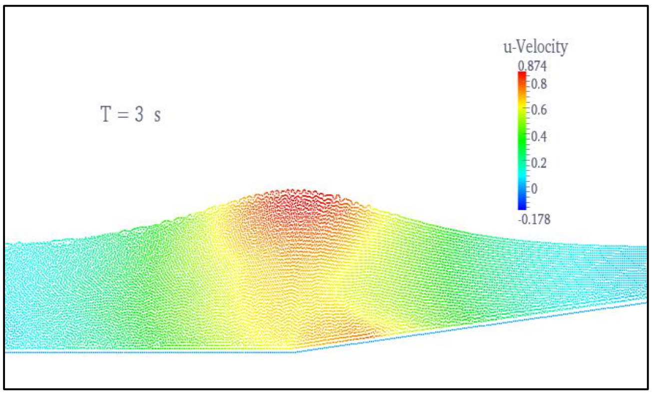
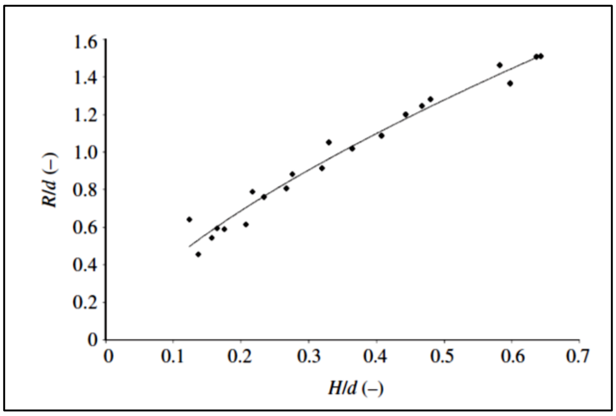


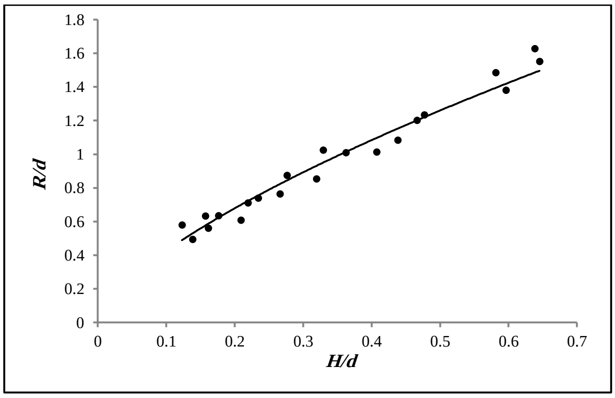
Figure 3. Schematic view of wave run-up velocity for H=0.155m and d=0. 26 and $\gamma = 0.107$ radian.

Table 1. Comparison between experimental data of Baldock et al. [26] and SPH model parameters and values. *d*, water depth; *H*, wave height; *R*, natural run-up elevation

case	d(cm)	H(cm)	H/d	R/d(Experimental – Data)	R/d(SPH – Model)
1	26	3.2	0.123	0.541	0.58
2	26	4.1	0.157	0.649	0.633
3	26	6.1	0.234	0.761	0.74
4	26	8.3	0.319	0.911	0.854
5	26	10.6	0.407	1.089	1.014
6	26	15.5	0.596	1.364	1.381
7	21	2.9	0.138	0.454	0.495
8	21	3.7	0.176	0.590	0.636
9	21	5.6	0.266	0.803	0.765
10	21	7.6	0.362	1.02	1.01
11	21	9.8	0.466	1.244	1.203
12	15.5	2.5	0.161	0.594	0.561
13	15.5	3.4	0.219	0.787	0.712
14	15.5	5.1	0.329	1.051	1.025
15	15.5	7.4	0.477	1.278	1.235
16	15.5	10	0.645	1.51	1.552
17	10.5	2.2	0.209	0.615	0.61
18	10.5	2.9	0.276	0.879	0.876
19	10.5	4.6	0.438	1.197	1.085
20	10.5	6.1	0.581	1.463	1.485
21	10.5	6.7	0.638	1.504	1.628



(a)



(b)

Figure 4. Experimental (a) and SPH model (b) values normalized run-up versus normalized wave height at the beach toe

5.2. Run-up Errors using SPH Method

The classical scaling of the run-up for solitary waves can be obtained as Eq. (5):

$$\frac{R}{d} = \alpha(\frac{H}{d})^{\beta} \quad (5)$$

where α and β are empirical parameters that depend on beach slope, breaking conditions and frictional effects [23, 28, 29]. Correlation coefficient R^2 obtained from experimental and SPH model are 0.96 and 0.958 respectively showing very good agreement (Figure 4). Finally, in this study, with solitary wave run-up data on beach slope (0.107 radian) and different wave depth and height, two relations were obtained for SPH model:

$$R/_{d} = 2.0111(H/_{d})^{0.6749} \quad (6)$$

$$R/_{d} = 0.3942(H/_{d})^{2} + 1.7071(H/_{d}) + 0.312 \quad (7)$$

The accuracy of these relations is shown in Table 2 and Figure 5. Clearly, it can be seen that the second relation (2) is more accurate. In comparison, the average errors by using the SPH model is about 4.528% that represents a good result of using SPH method.

Table 2. Error values obtained from SPH model and two relations in comparison with experimental results

Case	H/d	R/d (Experimental –Data)	R/d (SPH–Model)	Relation 6	Relation 7	SPH Model error	Relation.6 error	Relation.7 error
1	0.123	0.541	0.58	0.489	0.528	7.209	9.632	2.415
2	0.157	0.649	0.633	0.576	0.590	0.939	9.792	7.710
3	0.234	0.761	0.74	0.755	0.733	2.760	0.841	3.673
4	0.319	0.911	0.854	0.930	0.897	6.257	2.100	1.572
5	0.407	1.089	1.014	1.096	1.072	6.887	0.675	1.553
6	0.596	1.364	1.381	1.418	1.469	1.246	3.976	7.732
7	0.138	0.454	0.495	0.528	0.555	9.031	16.381	22.266
8	0.176	0.590	0.636	0.623	0.625	7.797	5.531	5.875
9	0.266	0.803	0.765	0.823	0.794	4.732	2.464	1.123
10	0.362	1.02	1.01	1.013	0.982	0.980	0.687	3.762
11	0.466	1.244	1.203	1.201	1.193	3.296	3.438	4.091
12	0.161	0.594	0.561	0.586	0.597	5.556	1.296	0.515
13	0.219	0.787	0.712	0.722	0.705	9.530	8.309	10.450

14	0.329	1.051	1.025	0.950	0.916	2.474	9.638	12.816
15	0.477	1.278	1.235	1.220	1.216	3.365	4.515	4.853
16	0.645	1.51	1.552	1.496	1.577	2.781	0.933	4.442
17	0.209	0.615	0.61	0.699	0.686	0.813	13.691	11.545
18	0.276	0.879	0.876	0.844	0.813	0.341	4.035	7.487
19	0.438	1.197	1.085	1.152	1.135	9.357	3.757	5.152
20	0.581	1.463	1.485	1.394	1.437	1.504	4.713	1.785
21	0.638	1.504	1.628	1.485	1.562	8.245	1.268	3.829
			Average errors			4.528	5.126	5.935

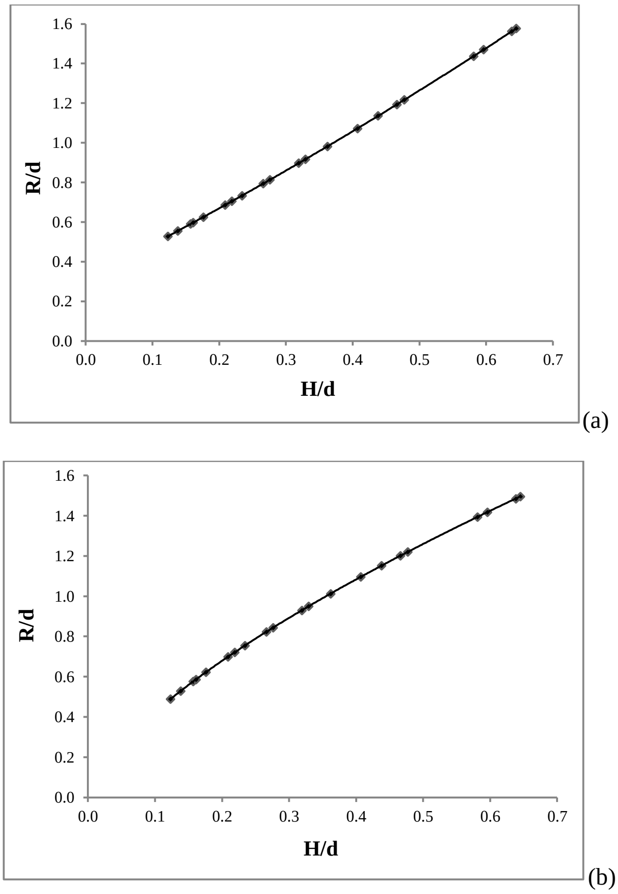


Figure 5. Diagram obtained from relations 6(a) and 7(b)

6. Wave Overtopping Modeling

In this section, the SPH model is employed to simulate a wave overtopping. The wave overtopping model is shown in Figure 6. The bathymetry comprised a 3m long horizontal section from the wave-maker to the toe of a uniform long sloping beach of gradient $\gamma = 0.107$ (Figure 6). According to [26], the sloping beach was constructed in two parts: a fixed lower section below the still water line (SWL), which is the position of the initial shoreline, and an adjustable beach with removable panels above the SWL. The origin of the horizontal coordinate is at the SWL and positive onshore. The surface of the beach was a smooth painted marine plywood bed. Joints between adjacent panels were sanded flush to minimize additional roughness. In wave overtopping modeling by SPH method the panel wasn't employed but the slope considered long enough to model the actual models well. The water depth over the horizontal section of the model was also varied between *d* =0.105–0.26 m; this additionally changed the beach truncation position relative to the SWL. The wave height of the model, *H*, varied between 0.022-0.155 m .The vertical distance between water level and top of the slope, *Z*, for *d*=0.21 m is 0.062 and 0.146 m and for *d*=0.26 m is 0.091 and 0.175 m.

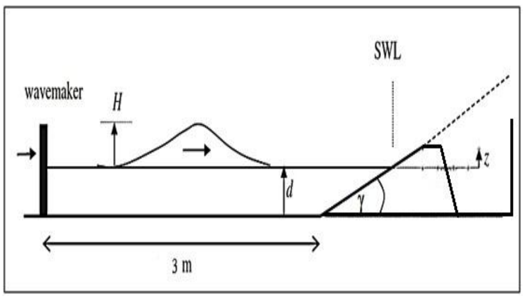


Figure 6. The initial geometry of solitary wave overtopping

6.1. Wave Overtopping Results using SPH Method

The mean wave overtopping volume is an important parameter in the study of wave overtopping. To estimate the wave overtopping volume for 17 cases, Baldock et al. [26] used experimental data of wave overtopping (Figure 7). In this study for summary, only the case number 13 is shown in Figure 7. In simulation, using SPH method for the case number 13 resulted that it took 10s for the wave to become stable. Therefore, for other cases the mean wave overtopping volume is calculated using the wave overtopping simulation between t=7s and t=12s. As it shown high wave overtopping velocities occur at the front of the wave overtopping crest which is quite reasonable. Figure 8 shows the schematic view of wave overtopping velocity.

Table 3 lists the experimental data of [26] and SPH model parameter and values for all the 17 cases. The experimental values were exported from [26]. The parameter d, H, and q represent water depth, wave height, and dimensional overtopping volume (liters per m width), respectively. The comparison between experimental data of [26] and SPH model is shown in Figure 9. It can be seen that SPH model diagram is in a good agreement with those via [26].

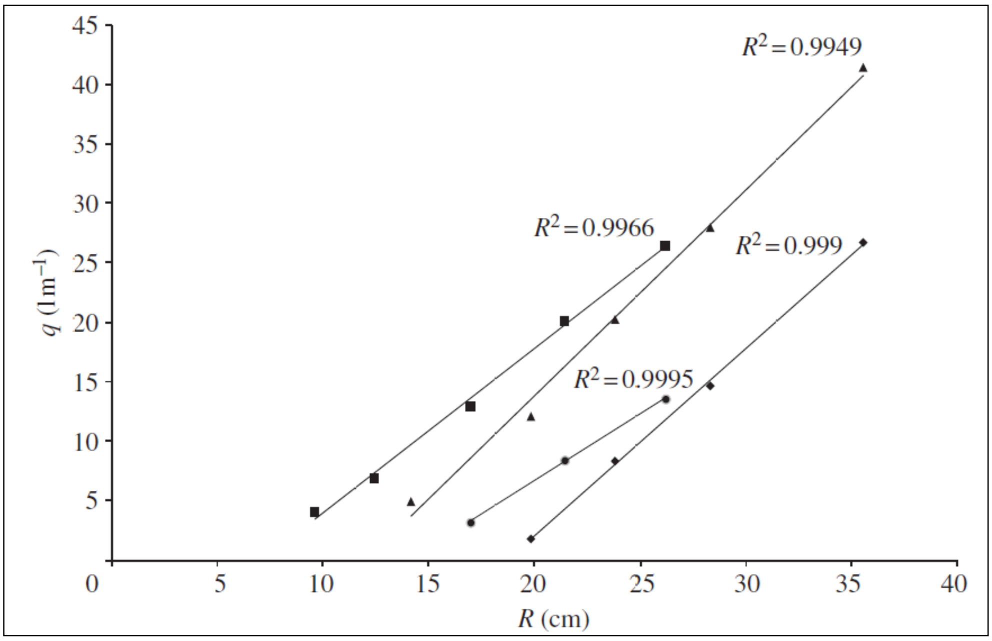
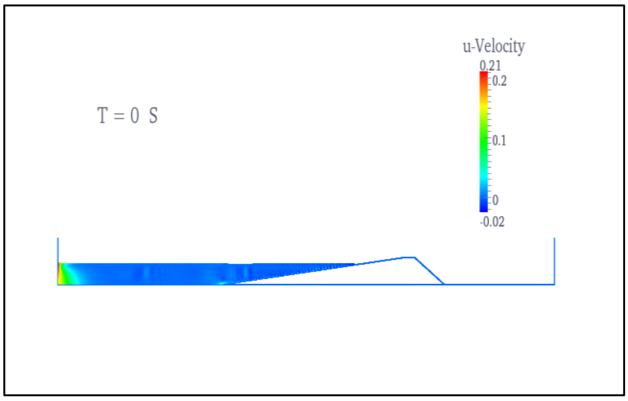


Figure 7. Dimensional wave overtopping volume versus wave overtopping elevation for varying water depths and truncation elevation (*d*, *z*), solitary waves. Filled squares, (21, 6.2 cm); filled circles, (21, 14.6 cm); filled triangles, (26, 9.1 cm); filled diamonds, (26, 17.5 cm)



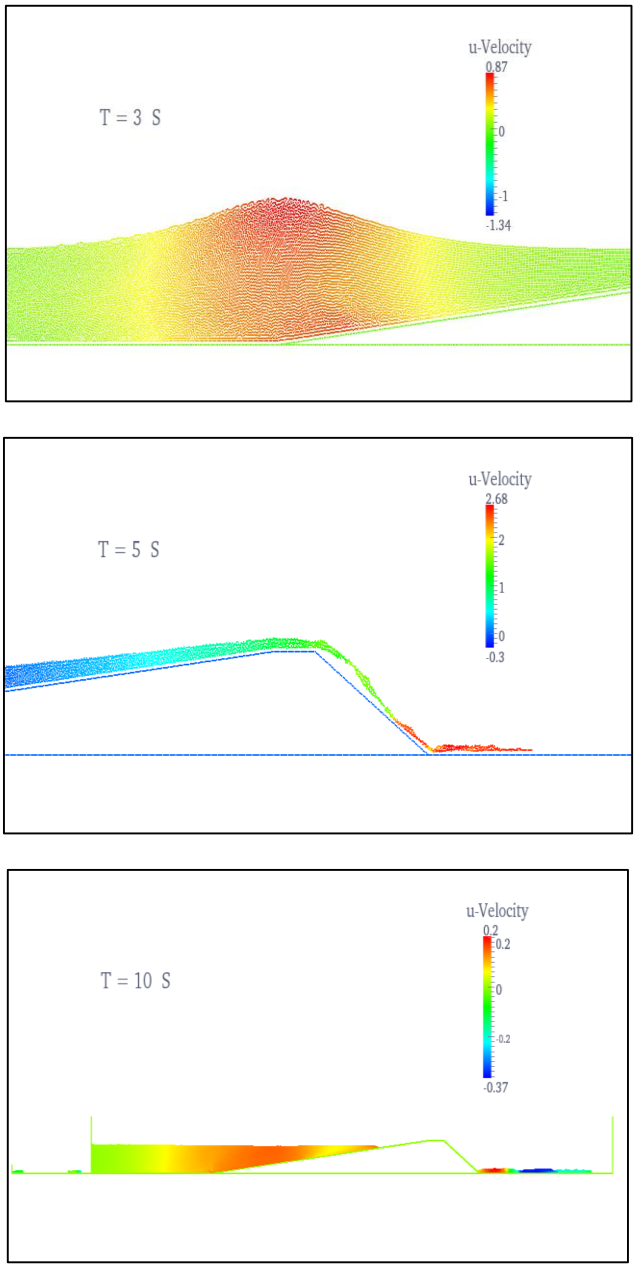
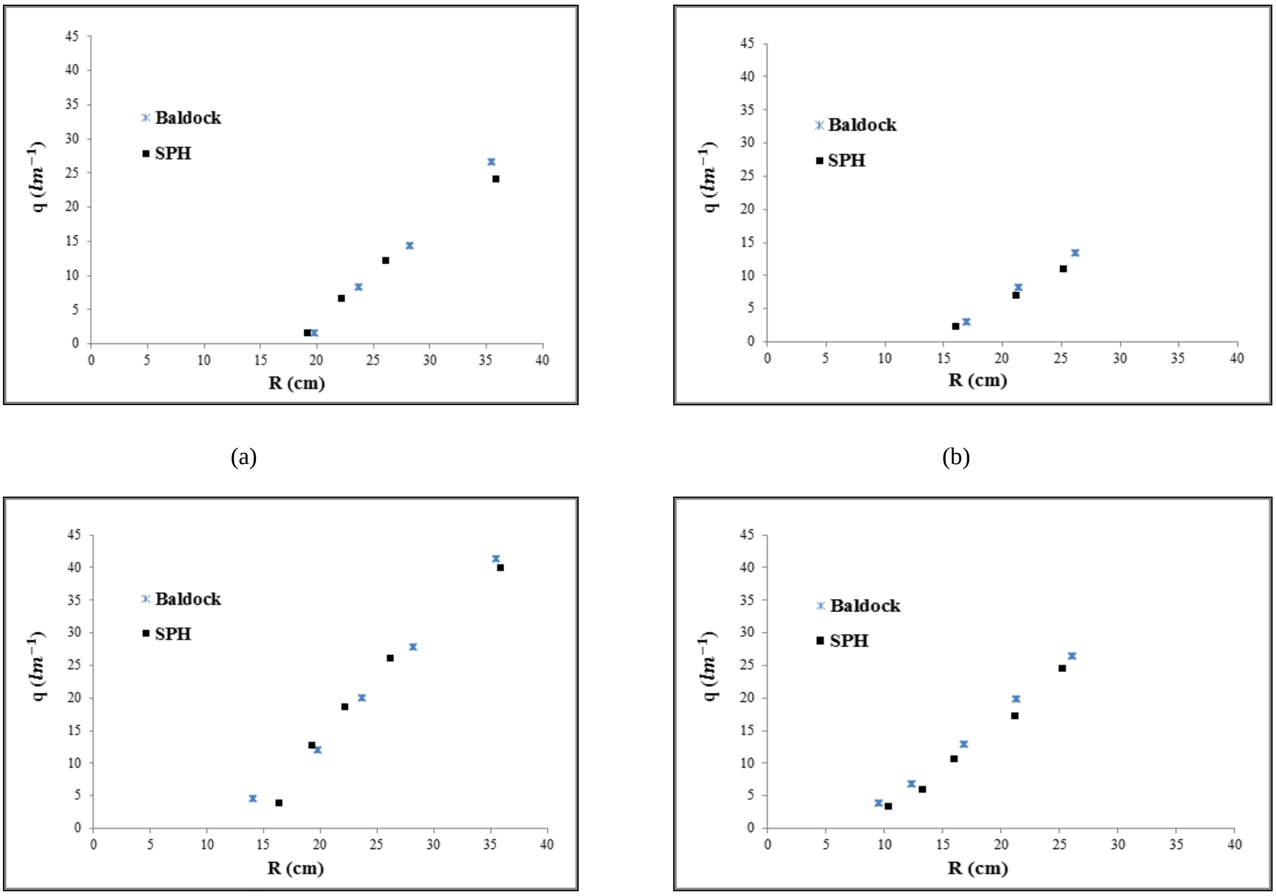


Figure 8. Schematic view of wave overtopping velocity for *H*=0.155 m and *d*=0. 26 and $\gamma = 0.107$ radian

Table 3. Comparison between experimental data of Baldock et al. [26] and SPH model parameters and values. *d*, water depth; *H*, wave height; *q*, dimensional overtopping volume

case	d(cm)	H(cm)	Z	R/d (Experimental–Data)	R/d (SPH – Model)
1	21	2.9	6.2	3.754	3.276
2	21	3.7	6.2	6.673	5.904
3	21	5.6	6.2	12.782	10.548
4	21	7.6	6.2	19.836	17.208
5	21	9.8	6.2	26.361	24.408
6	21	5.6	14.6	3.023	2.196
7	21	7.6	14.6	8.196	6.912
8	21	9.8	14.6	13.332	10.944
9	26	4.1	9.1	4.462	3.888
10	26	6.1	9.1	11.896	12.672
11	26	8.3	9.1	19.923	18.504
12	26	10.6	9.1	27.802	25.992

13	26	15.5	9.1	41.303	39.924
14	26	6.1	17.5	1.448	1.476
15	26	8.3	17.5	8.196	6.588
16	26	10.6	17.5	14.347	12.096
17	26	15.5	17.5	26.648	24.084



(a) (b) (c) (d)

Figure 9. Wave overtopping volume computed by SPH method in comparison with experimental data obtained from experimental data of [27]. (a): (21, 6.2 cm); (b): (21, 14.6 cm); (b): (26, 9.1 cm); (c): (26,17.5 cm)

7. Conclusions

The SPH method, with its Lagrangian formulation, provides a methodology for the detailed examination of water waves. It is particularly suited to those cases where there is splash, or flow separation, as the determination of the free surface is not difficult. In this study shows that the SPH method can even provide good quantitative predictions and permit one-to-one comparisons between numerical and experimental results. The wave profiles generated by the SPH method are in good quantitative agreement with the experimental ones, both in phase and amplitude. To simplify the overtopping phenomenon, the study was restricted to two-dimensional waves. The study showed that

During the modeling, as the number of particles increased (by minimizing the distances between particles) and the time-step decreased, the accuracy of modeling increased. It observed that the computing time increased and took more than 4 times. On the other hand, computing showed that 0.006 m was the optimum size of particles.

Approximately the error for 54% of the cases modeled for wave run-up was less than 4%. Moreover, the errors for all cases were less than 10% showing acceptable results. The results indicate that relatively large prediction

The correlation coefficient R^2 obtained about 1 for both relations represented in the manuscript (relation 6 and 7). A time series of the wave overtopping volume shows a high peak, when the front of the wave overtopping crest passes. Therefore, high wave overtopping velocities occur at the front of the wave overtopping tongue. This issue is quite reasonable. The results of wave run-up estimated the correlation coefficient for different cases about 0.96. This sequence demonstrates that the model is unified for different cases of water depth and has identical condition.
The free surface flow in modeling problems was considered with high accuracy. Good agreement between experimental data and numerical results confirms this result. Thus it can be observed that SPH method can accurately model free surface flow with high variations. It should be noted that particles are minified to 0.006m, and in some models more

than 30000 particles are present which have made the results more accurate. Obtained results show that the present SPH model is a convenient model for simulating complex fluid mechanics problems with free surface boundary conditions.

Acknowledgements

The authors would like to show their gratitude to Mr. Baldock et al. for their comprehensive and freely database of wave overtopping and wave run-up.

References

[1] C.E. Synolakis, E.N. Bernard, Tsunami science before and beyond Boxing Day, Philosophical Transactions of the Royal Society A-Mathematical Physical and Engineering Sciences 364 (2006) 2231–2265.
[2] J.C. Borrero, Field data and satellite imagery of tsunami effects in Banda Aceh, Science 308 (2005) 1596.
[3] T. Pullen, N.W.H. Allsop, T. Bruce, A. Kortenhaus Schuttrumpf, H. van der Meer J.W. EurOtop wave overtopping of sea defences and related structures: Assessment Manual, Eds (2007).
[4] M. Umeyama, Wave overtopping on vertical boundary and water–surface displacement. ASCE, Journal of Waterway, Port, Coastal, and Ocean Engineering 119 (1993) 243–260.
[5] T.S. Hedges, M.T. Reis, Random wave overtopping of simple sea walls, a new regression model. Proceedings of the Institution of Civil Engineers, Water and Maritime Engineering 130 (1998) 1–10.
[6] S. Marrone, A. Colagrossi, M. Antuono, C. Lugni, M.P.A. Tulin, 2D+t SPH model to study the breaking wave pattern generated by fast ships. Journal of Fluids and Structures 27 (2011) 1199–1215.
[7] D.T. Cox, C.P. Scott, Exceedance, probability for wave overtopping on a fixed deck, Ocean Engineering 28 (2001) 707–721.
[8] D.T. Cox, J.A. Ortega, Laboratory observations of green water overtopping a fixed deck, Ocean Engineering 29 (2002) 1827–1840.
[9] M.J. Walkden, D.J. Wood, T. Bruce, Peregrine D.H. Impulsive seaward loads induced by wave overtopping on caisson breakers, Coastal Engineering 42 (2001) 257–276.
[10] A. Losada, J.J. Losada, Smoothed particle hydrodynamics: theory and application to non-spherical stars, Monthly Notices of the Royal Astronomical Society 181 (1977) 375–389.
[11] W. Benz, E. Asphaug, Impact simulations with fracture. I.Methods and tests, Icarus 107 (1994) 98–116.
[12] W. Benz, E. Asphaug, Simulations of brittle solids using smoothed particle hydrodynamics, Computer Physics Communications 87 (1995) 253-265.
[13] J.J. Monaghan, Gravity currents and solitary waves, Physica D 98 (1996) 523–533.
[14] J.J. Monaghan, A. Kos, Solitary waves on a Cretan beach. Journal of Waterway, Port, Coastal, and Ocean Engineering 125 (1999) 145–154.
[15] J.J. Monaghan, Simulating free surface flows with SPH, Journal of Computational Physics 110 (1994) 399–406.
[16] J.J. Monaghan, A. Kos, Scott Russell's wave generator, Physics of Fluids 12 (2000) 622–630.
[17] R.A. Dalrymple, O. Knio, SPH Modeling of water waves, Coastal Dynamics 1 (2001) 779–787.
[18] Reis, A numerical approach to the testing of fusion process, Astronomical Journal 82 (1977) 1013–1024.
[19] M. Gomez-Gesteira, R. Dalrymple, Using a 3D SPH method for wave impact on a tall structure, Journal of Waterway, Port, Coastal, and Ocean Engineering 130 (2004) 63–69.
[20] A. Soliman, Numerical study for small freeboard wave overtopping and overflow of sloping sea walls. PhD thesis, The University of Nottingham, United Kingdom, (2003).
[21] A. Soliman, D.E. Reeve, Numerical study for small negative freeboard wave overtopping and overflow of sloping sea walls. Proc. Coastal Structures, Portland USA, (2004) 643–655.
[22] N. Garcia, J.L. Lara, I.J. Losada 2-D numerical analysis of near-field flow at low-crested permeable breakwaters, Coastal Engineering 51 (2004) 991–1020.
[23] Y. Li, F. Raichlen, Non-breaking and breaking solitary wave run-up, Journal of Fluid Mechanics 456 (2002) 295–318.
[24] J.J. Monaghan, A. Rafiee, A simple SPH algorithm for multi-fluid flow with high density ratios, Int. J. Numer. Methods Fluids 71 (2013) 537–561.
[25] J.J. Reis, Smoothed particle hydrodynamics, Annual Review of Astronomy and Astrophysics (1992) 543–574.
[21] H. Yeh, Maximum fluid forces in the tsunami run-up zone, ASCE, Journal of Waterway Port Coastal and Ocean Engineering 132 (2006) 496-500.
[26] T.E. Baldock, D. Peiris, A.J. Hogg, Overtopping of solitary waves and solitary bores on a plane beach. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 468 (2012) 3494–3516.
[27] C.E. Synolakis, The run-up of solitary waves, Journal of Fluid Mechanics 185 (1987) 523–545.
[28] A.G. Borthwick, L. Ford, M. Weston, B.P. Taylor, P.H. Stansby, P.K Solitary, wave transformation, breaking and run-up at a beach. Proceedings of the ICE - Maritime Engineering 159 (2006) 97–105.
[29] P.A. Reis, H.A. Reis, Analytical solutions for tsunami run-up on a plane beach: single waves, N-waves and transient waves, Journal of Fluid Mechanics 645 (2010) 27–57.