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# Translation Invariant Wavelet Based Noise Reduction Using a New Smooth Nonlinear Improved Thresholding Function

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**Keywords**

Image de-noising,
Improved thresholding function,
Translation invariant wavelet transform,
Noise reduction.

**Abstract**

In this paper, a new type of thresholding function is introduced for wavelet based image de-noising. Here we combined the new smooth nonlinear improved thresholding function with translation invariant wavelet transform (TIWT). Unlike the common thresholding functions (hard and soft thresholding), the new proposed function is smooth and nonlinear. Applying this thresholding function on wavelet transform provides us with better resolution and higher peak signal to noise ratio (PSNR) in comparison with some other available techniques in image de-noising. The proposed method achieves up to 2.45 dB improvement over the state-of-the-art for de-noising 'Boat' image.

## 1. Introduction

Noise is unwanted information which can contaminate the image during its transmission and reception processes. Therefore noise, reduction is needed to analyze the images properly. The main focus in image de-noising is keeping the most important information and discarding non-important components.

Recently, researchers are focusing on wavelet based noise reduction. This wavelet transform can also be combined with different thresholding functions. Ideal spatioal adaption by wavelet shrinkage [1] and adapting to unknown smoothness via wavelet shrinkage [2] are proposed by Donoho and Johnstone in 1993 and 1995, respectively. Adaptive image de-noising using wavelet thresholding is introduced by Dong [3]. Zhang proposed hyper-spectral image de-noising with cubic total variation model [4]. Zhang [5] proposed the space-scale adaptive noise reduction in images based on thresholding neural network and TNN for adaptive noise reduction [6]. Chang et al., in 2000 introduced adaptive wavelet thresholding for image de-noising and compression [7]. Image de-noising in wavelet domain using a new thresholding function was proposed by Norouzzadeh and Rashidi [8]. Hyper-spectral image de-noising using 3D wavelets is proposed in 2012 by Rasti et al., [9]. Golilarz and Demirel proposed a new improved thresholding function for wavelet based noise reduction [10]. Sendur and Salesnick proposed bivariate shrinkage for wavelet based de-noising exploiting inter scale dependency [11]. An Efficient SVD-based method for image de-noising is introduced in a study conducted by Guo et al., in 2016 [12]. De-noising and dimensionality reduction of hyper-spectral images using framelet transform with different shrinkage functions is proposed in 2016 [13]. Chen and Qian conducted de-noising of hyper-spectral imagery using principal component analysis and wavelet shrinkage [14].

Wavelet image de-noising based on improved thresholding neural network and cycle spinning is introduced by Sahraeian et al., [15]. Moreover, the contourlet transform for image de-noising using cycle spinning is proposed by Eslami and Radha [16].

Image de-noising using orthogonal wavelet transform sometimes causes visual artifacts called Gibbs phenomena [17]. Therefore, Coifman and Donoho introduced translation invariant image de-noising to enhance the visual inspection and PSNR values [17].

In this paper, the authors proposed to utilize translation invariant wavelet transform (TIWT) combined with a new type of smooth nonlinear thresholding function. This technique is introduced to improve the visual quality and PSNR value and reduce the effects of Gibbs phenomena caused by using orthogonal wavelet transform. Experimental results show the superiority of the proposed method over some other available techniques in terms of acquiring higher PSNR value and better visual quality.

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## 2. Discrete Wavelet Transform (DWT)

We can write *u(t)* in terms of scale function $\varphi(\mathrm{t})$ and wavelet function $\rho(\mathrm{t})$ [1]. One dimensional discrete wavelet transform (DWT) can be written as following

$$u(t) = \sum_k AC_{i0,k}\ \varphi_{i0,k} + \sum_{i\leq i_0}\sum_k DC_{i,k}\ \rho_{i,k} \qquad (1)$$

where $\varphi_{i,k}=2^i\varphi(2^it-k)$ is the scale function and $\rho_{i,k}=2^{i/2}\rho(2^it-k)$ stands for the wavelet function. Moreover, the inner products $AC_{i,k}$=<*u*, $\varphi_{i,k}$>, $DC_{i,k}$=<*u*, $\rho_{i,k}$> are scaling and wavelet coefficients, respectively.

In the discrete wavelet transform, the original signal *x* can pass through low pass *g(n)* and high pass *h(n)* filters. This process is followed by decimation (down sampling by 2) to get the approximation coefficients (*AC*) and detail coefficients (*DC*). Figure 1 shows the general block diagram of DWT.

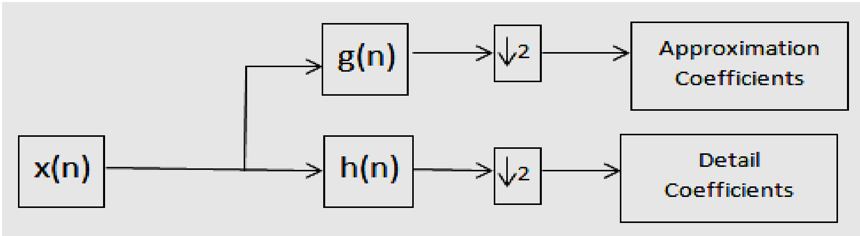
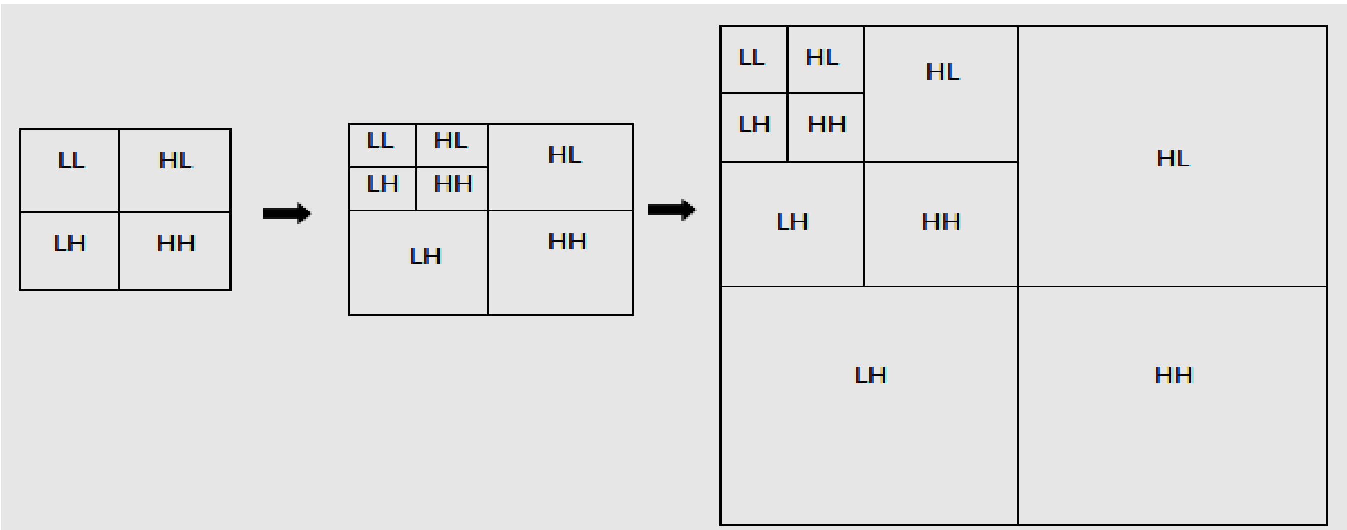



**Figure 1.** General Block Diagram of DWT

Low pass filtering provides us with losing all high frequencies. Hence, half of the information will be discarded and the resolution will be halved as a consequence. Higher level of decomposition will take place in approximation coefficients or low frequency sub band. For the second level of decomposition, the output of *g(n)* will pass through low pass and high pass filters again with down sampling by 2 which is also followed by losing high frequencies at level 2 and the resolution will be halved of the previous level. For higher levels of decomposition we have the same process again.

In the two dimensional discrete wavelet transform (2D-DWT) and for first level of decomposition, we apply 1D-DWT on rows and columns [9] to obtain four sub images HH, HL, LH and LL as can be seen in Figure 2(a). Higher level of decomposition for 2D-DWT is also possible which Figures 2(b)-(c) show 2 and 3 levels of decomposition for 2D-DWT.




(a) (b) (c)

**Figure 2.** (a) First level, (b) Second level and (c) Third level of decomposition for 2D-DWT [10]

## 3. Noise Reduction Using Wavelet Transform

The main objective in wavelet based image de-noising is discarding the noise from image. Applying the DWT on the input noisy image provides us with wavelet coefficients including small noisy components and detail or important coefficients. These detail coefficients are carrying the most important characteristics of image which should be kept while those small noisy coefficients causing low resolution should be removed or discarded. Doing this process required a proper thresholding function (which should be fitted to the wavelet coefficients properly) and a suitable threshold value derived from data. Figure 3 shows general block diagram for wavelet based image de-noising. As we see in this figure, employing 2D-DWT on input noisy image gives four sub images HH, HL, LH and LL having high and low frequencies. Then thresholding function can be applied on these sub bands to threshold wavelet coefficients with a defined threshold value. Last step is reconstructing the output de-noised image by applying inverse discrete wavelet transform (IDWT) on these thresholded wavelet coefficients.

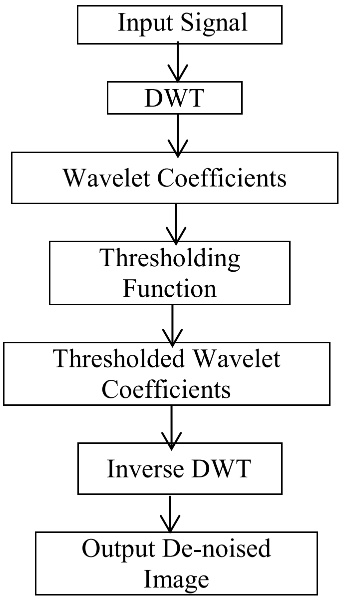



**Figure 3.** General block diagram of de-noising technique [10]

## 4. Thresholding Techniques

Threshold is a defined ideal verge or value which helps us to keep the desired components (large crucial coefficients) and discard unwanted or undesired coefficients (small noisy coefficients). Performing this process is called 'thresholding'. For image de-noising we can also use thresholding techniques. Using this concept, we have the following common thresholding functions.

### *4.1. Hard Threshold*

In the hard thresholding, we keep those wavelet components which their absolute values are greater than threshold value and kill others. Eq. (2) denotes general formula for hard thresholding function [3].

Figure 4 illustrates the hard thresholding function. As we see in this figure, there is a discontinuity in the threshold value. To solve this problem, we can shrink the coefficients by the threshold value as we can see in the same figure.

$$f_h\,(x) = \begin{cases} x\,, & |x| > t \\ 0\,, & otherwise \end{cases} \tag{2}$$

where $f_h\,(x)$ is hard thresholding function, $x$ is wavelet coefficients and $t$ is universal threshold [1].

### *4.2. Soft Threshold*

In soft thresholding we shrink those wavelet components which their absolute values are greater than the threshold value and kill others. Eq. (3) denotes the general formula for soft thresholding function [3]. Figure 4 shows soft thresholding function.

$$f_s\,(x) = \begin{cases} sgn(x).\left(|x| - t\right), & |x| \geq t \\ 0\,, & otherwise \end{cases} \tag{3}$$

where $f_s\,(x)$ is the soft thresholding, $x$ is wavelet coefficients and $t$ is the universal threshold [1].

The universal threshold can be obtained by Eq. (4) as

$$t = \sigma_n \sqrt{2log(d)} \tag{4}$$

in which $d$ is the number of pixels and $\sigma_n$ denotes the standard deviation of noise [1] which is given by Eq. (5)

$$\sigma_n = Median\,(|HH|)/0.6745 \tag{5}$$

where $HH$ is the wavelet detail coefficients in diagonal direction [3].

## 5. Translation Invariant Wavelet Transform

Translation variance and poor directional properties are some disadvantageous of using DWT [9]. To overcome this problem, it is suggested to utilize un-decimated wavelet transform (UWT). In this wavelet transform, original signal can also pass through high pass and low pass filters but in spite of the DWT, this process is not followed by decimation. We do not have down sampling or sub sampling here. Then the length of output signal after each level of decomposition is the same as original input, so we are not losing any information or resolution by doing more levels of decompositions. For this reason, it is called un-decimated wavelet transform. This wavelet transform is also known as translation invariant wavelet transform (TIWT) [9, 11]. In this paper, we used UWT with proposed smooth nonlinear improved thresholding function for image de-noising.

The general formula of a noisy image is given in Eq. (6). Here our aim is minimizing the *MSE* which is given in Eq. (7)

$$y = x + n \tag{6}$$

where $y$ is input noisy image, $x$ is input noise free signal and $n$ is additive white Gaussian noise (AWGN) with zero mean and standard deviation of $\sigma$.

$$MSE = \frac{1}{N^2}\sum_{m,p=1}^{N}(\hat{x}(m,p) - x(m,p))^2 \tag{7}$$

where $N^2$ is the number of pixels and $\hat{x}$ is output de-noised image.

## 6. Proposed Thresholding Function

In this part a new type of thresholding function is introduced to be combined with UWT for image de-noising. This function is nonlinear, continuous and data dependent. Eq. (8) indicates improved thresholding function and Figure 4 shows the alternative thresholding functions including hard, soft and proposed smooth nonlinear function in one graph.

$$f(x) = \begin{cases} sign(x)\left[|x| - cos\left(\frac{\pi}{5}\left|\frac{t}{x}\right|\right)t\right], & |x| > t \\ 0.19\frac{x^3}{t^2} & , |x| < t \end{cases} \tag{8}$$

where $x$ is wavelet coefficients and $t$ is threshold value.

## 7. Experimental Results and Discussion

In this part, two experiments are used to compare the performance analysis of the proposed method with some other alternative techniques available in the literature. In these experiments, 'db4' with four levels of decomposition is used. In the first experiment, a comparison between the proposed function with Noorbakhsh's improved thresholding function [10] in the universal threshold case was conducted. Here 'Cameraman', 'Peppers', 'Boat' and

'Gold Hill' images (256×256) contaminated by additive white Gaussian noise (AWGN) with zero mean and standard deviation of 20 were utilized. Figure 5 shows comparison of visual inspection and PSNR values between the proposed nonlinear thresholding function and Noorbakhsh's function [10].

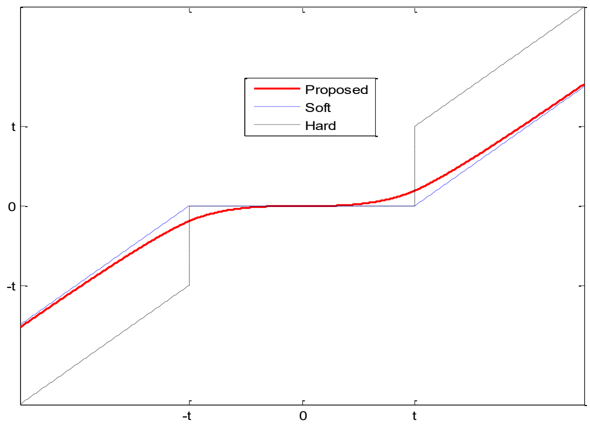



**Figure 4.** Hard, soft and proposed thresholding functions

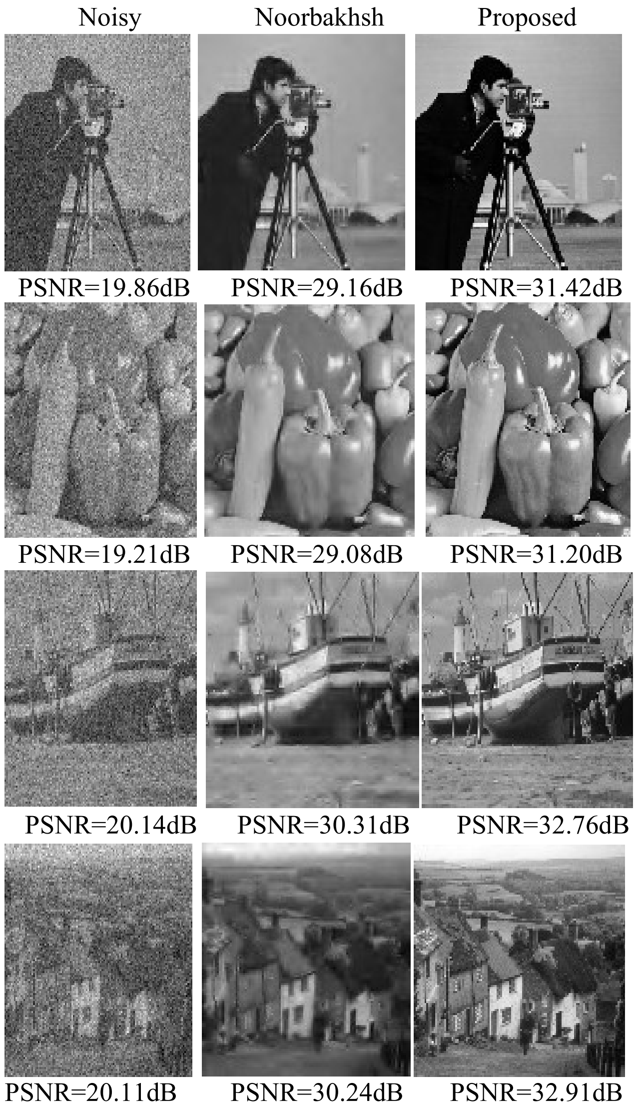



**Figure 5.** Visual quality comparison of the proposed function (right column) with noisy image (left column) and Noorbakhsh's improved thresholding function (middle column) [10] in the universal case

In the second experiment, the performance analysis of the proposed method is compared with Bayes shrink [10], Eslami's method [16] and Norouzzadeh's proposed wavelet based noise reduction function (in universal threshold case) [8]. Here, we used (256×256) 'Barbara' and band 20 of 'Indian Pine' hyper-spectral images corrupted by AWGN with zero mean and standard deviation of $\sigma$. Data set for Indian Pine hyper-spectral image is available in [9]. Figures 6 and 7 show PSNR comparison between the proposed method with some other available state-of-the-art de-noising techniques for changing different $\sigma$ values.

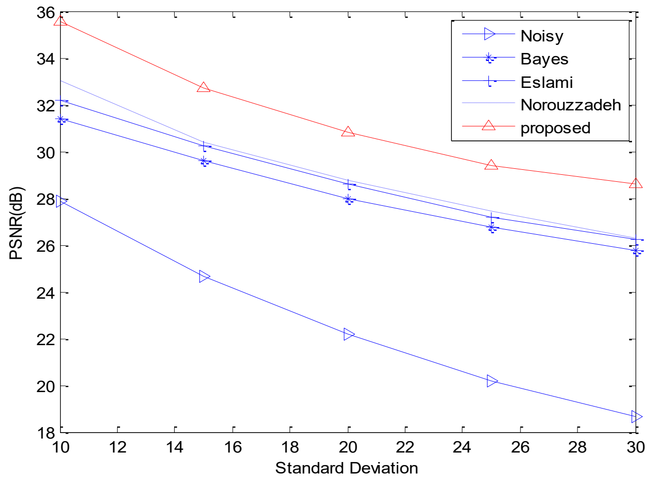



**Figure 6.** PSNR results versus standard deviation for different de-noising methods for 'Barbara' image

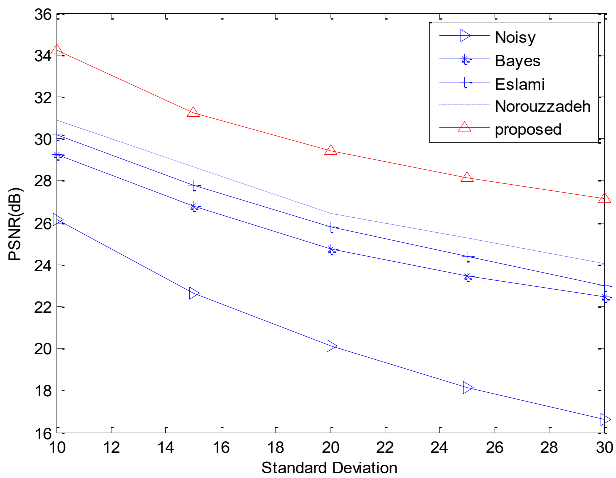



**Figure 7.** PSNR results versus standard deviation for different de-noising methods for band 20 of 'Indian Pine' hyper-spectral image

## 8. Conclusions

In this paper, a new type of smooth nonlinear improved thresholding function is introduced to combine with un-decimated wavelet transform (UWT) for image de-noising. One of the advantageous of the proposed thresholding function over standard hard and soft thresholding is its smoothness and nonlinearity. This function is data dependent as well. Applying the proposed function on un-decimated wavelet transform will result in improving the visual quality and obtaining higher Peak Signal to Noise Ratio (PSNR) in comparison with some other alternative techniques available in the literature.

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