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Research Article

Diffusion from a point source using combination between logarithmic and power laws of wind speed

M. Abdel-Wahab[a], Khaled S. M. Essa[b], M. Embaby[b] and Sawsan E. M. Elsaid[b]

[a] Meteorology and Astronomy Department, Faculty of Science, Cairo, University
[b] Mathematics and Theoretical Physics, NRC, Atomic Energy Authority, Cairo- Egypt

* Corresponding author. Tel.: (002) 01113975756.; fax: (002)44620790
E-mail address:mohamedksm56@yahoo.com.

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A b s t r a c t

In the present paper, a model for the diffusion of material from a point source in an urban atmosphere is incorporated. The plume is assumed to have a well-defined edge at which the concentration falls to zero. The vertical wind shear is estimated using combination between logarithmic and power laws under different stability conditions. Diffusion and advection of conservative material as it travels downwind is calculated. The concentrations estimated from this model were compared favourably with the field calculated of other investigators such as power and logarithmic law models. Also we calculate the concentration at the ground of the Iodine (I_{138}) which agrees with the observed concentration value after adjusted its source strength.

1. Introduction

Conventional Gaussian plume models are commonly used for the air quality analysis and regulatory purposes. These Gaussian models are based on the solution of an advection diffusion equation derived by assuming that the wind speed and eddy diffusivity don't have spatial and temporal variations [1].

The diffusion from a point source using logarithmic law of the wind speed is studied by [2] and the diffusion from a point source using power law of the wind speed is studied by [3].

The purpose of this study is to suggest a simple physical realistic model which depends on the surface roughness of the ground area over which diffusion of pollution takes place; also, wind speeds are taken as a combination between logarithmic and power laws in different stability classes in the atmosphere. This definitely more closely represents real life situations than does treating wind as a constant quantity. The conservative material as it travels downwind is investigated.

2-Proposed model and its components

A point source normal to mean wind direction with height “h” situated at the ground level and emission rate, the equation in two dimensions homogeneity in the lateral direction is assumed. Fig. 1 describes the coordinate system direction of the mean wind. The effective height denoted by $H=h_s + \Delta h$, where h_s is the stack height and Δh is the plume height which increases as the plume travel downwind. The analysis that follows assumes steady-state conditions; that is, the variables of interest; for example the mean wind speed, stability of the atmosphere, source strength, etc. don’t change in the time interval of interest. The ground surface is treated as a complete reflector of matter; that is, no removal occurs.

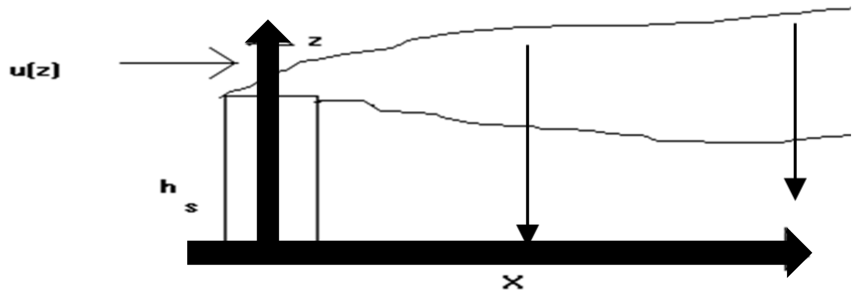

Fig. 1. Coordinate system direction of the mean wind.

3-The effective height

Defining the plume height Δh of diffusing matter as the distance from the stack height h_s to the point at which concentration has fallen to one-tenth of the surface value. The plume height has been calculated adopting the following equation [4].

$$\Delta h = 3(w/u)D_1$$

Where w is the exit velocity of the pollutants, and D_1 is the internal stack diameter. The effective stack height H equals:

$$H = h_s + \Delta h = h_s + 3\,(w/u)D_1$$

4-Mathematical derived

The conservation of mass equation with steady state can be written as:

$$Q = \int_{0}^{H} \overline{U}(z)\, C(z)\, dz \tag{1}$$

And assuming that the concentration takes the form of power series in z/H

$$\frac{C(z)}{C_0(z)} = 1 + \alpha_1\left(\frac{z}{H}\right) + \alpha_2\left(\frac{z}{H}\right)^2 + ... \tag{2}$$

where:

C = averaged concentration (Bq/m^3).

Q = actual emission rate of the point source (Bq)

α_1, α_2, ….. are constants

$\overline{U}(z)$ = average plume velocity (m /s).

z= plume height in meters (m).

C_0 (z) is the concentration at the edge of the plume (Bq/m^3) at which the concentration tends to zero.

H is the effective height of the plume (m).

The number of terms in the above series will depend upon desired goodness of fit to the observed data as shown in Fig. (2). It was found from the figure that the series in Eq. (2) gives a fairly good fit to observed data even if the first two terms are retained; that is

$$\frac{C(z)}{C_0(z)} = 1 + \alpha\left(\frac{z}{H}\right) \tag{3}$$

This equation is a straight line. The value "α" will depend upon the concentration desired at the edge of the plume. If the concentration at edge of the plume is defined as having r percent of the concentration then,

$\alpha = -1 + 0.01\, r$ and if $r = 0$ (3a)

$C/C_o = 1-(z/H)$ (3b)

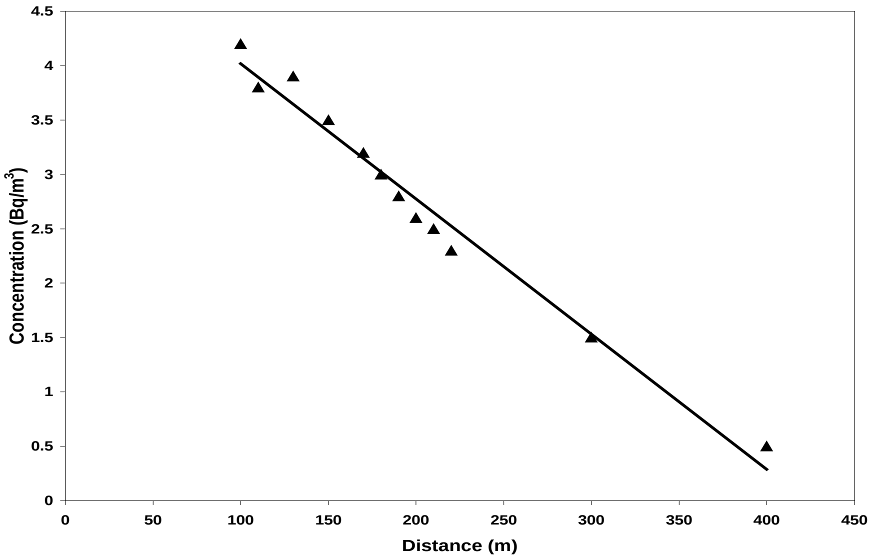

Fig.(2), The variation of the concentration of Iodine(I_{138}) with distance from the reactor

4.1 In neutral case:

The wind speed takes the form as follows:

$$\bar{U}(z) = U\left(\frac{\ln\left(\frac{z+z_0}{z_0}\right)}{\ln\left(\frac{z_1+z_0}{z_0}\right)}\right) \tag{a}$$

where:

z_o is roughness height in urban area (m).

U is the observed wind speed (m/s).

z_1 is the height of plume at 10m.

4.2 In stable case:

The shape of the wind speed is in the form, taking L >0 (L=55m) and β=5 as follows:

$$\bar{U}(z) = U\left(\frac{\ln\left(\frac{z+z_0}{z_0}\right)+\beta\left(\frac{z_1-z_0}{L}\right)}{\ln\left(\frac{z_1+z_0}{z_0}\right)+\beta\left(\frac{z_1+z_0}{L}\right)}\right) \tag{b}$$

4.3 In unstable case:

The wind speed takes the form, taking $L < 0$ (L= -2.5m) and $\beta=15$ as follows:

$$\bar{U}(z) = U\left(\frac{\ln\left(\frac{z+z_0}{z_0}\right)+\beta\left(\frac{z_1-z_0}{L}\right)}{\ln\left(\frac{z_1+z_0}{z_0}\right)+\beta\left(\frac{z_1+z_0}{L}\right)}\right) \quad \text{(c)}$$

Neutral case

Substituting from equations (a), (3b) into equation (1) and integrating we obtain that:

$$\frac{C_0}{Q} = \left(\frac{\ln\left(\frac{z_1+z_0}{z_0}\right)}{U}\right)\left[\frac{\left(2H^2+4Z_0H+2Z_0^2\right)\left(\ln\left(\frac{H+Z_0}{Z_0}\right)\right)-3H^2-2Z_0H}{4H}\right]^{-1}$$

(3)

Stable case

Substituting from equations (b), (3b) into equation (1) and integrating we obtain:

$$\frac{C_0}{Q} = \left(\frac{\ln\left(\frac{z+z_0}{z_0}\right)+\beta\left(\frac{z_1-z_0}{L}\right)}{U}\right) \left[\frac{\left(66H^2+132z_0H+66z_0^2\right)\left(\ln\left(\frac{H+z_0}{z_0}\right)\right)-66z_0H-99H^2-6z_0H^2+2H^3}{132H}\right]^{-1} \quad (4)$$

Unstable case

Substituting of equations (c), (3b) into equation (1) and integrating we obtain that:

$$\frac{C_0}{Q} = \left(\frac{\ln\left(\frac{z+z_0}{z_0}\right)+\beta\left(\frac{z_1-z_0}{L}\right)}{U}\right) \left[25\left(\frac{\left(2.0H^2+4.0z_0H+2.0z_0^2\right)\left(\ln\left(\frac{H+z_0}{z_0}\right)\right)-2.0z_0H-3.0H^2+12.0z_0H^2-4.0H^3}{H}\right)\right]^{-1} \quad (5)$$

Table1. Wind speed, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate in different stability condition.

U (m/s)	Δh(m)	H (m)	Co/Q*103 (sec/ m^3) in neutral	Co/Q*103 (sec/ m^3) in stable	Co/Q*105(sec / m^3) in unstable
5.27	2.28	45.28	7.72	7.66	55.75
5.31	2.26	45.26	7.66	8.75	63.43
5.34	2.25	45.25	7.62	8.39	60.86
6.37	1.88	44.88	6.47	6.97	50.87
5.17	2.32	45.32	7.86	8.98	65.01
4.45	2.7	45.7	9.02	9.27	66.99
5.1	2.35	45.35	7.96	9.11	65.91
4.81	2.49	45.49	8.4	9.55	68.93
4.81	2.26	45.26	8.4	9.2	66.53
4.86	2.47	45.47	8.32	8.77	63.57
5.36	2.24	45.24	7.6	8.4	60.99
5.19	2.31	45.31	7.83	8.59	62.32
4.41	2.22	45.22	9.09	8.88	64.28
5.54	2.17	45.17	7.37	8.42	61.12
5.2	2.31	45.31	7.81	9.5	68.6
5.61	2.14	45.14	7.28	8.28	60.09
5.79	2.07	45.07	7.07	7.86	57.2
6.27	1.91	44.91	6.56	7.85	57.08
5.93	2.02	45.02	6.92	8.35	60.6
6.01	2	45	6.83	8.19	59.46
5.41	2.22	45.22	7.53	8.5	61.65
5.75	2.09	45.09	7.12	8.65	62.73
5.26	2.28	45.28	7.73	9.05	65.46

Table (1), shows calculated values of U, Δh, H and Co/Q in different stability conditions.

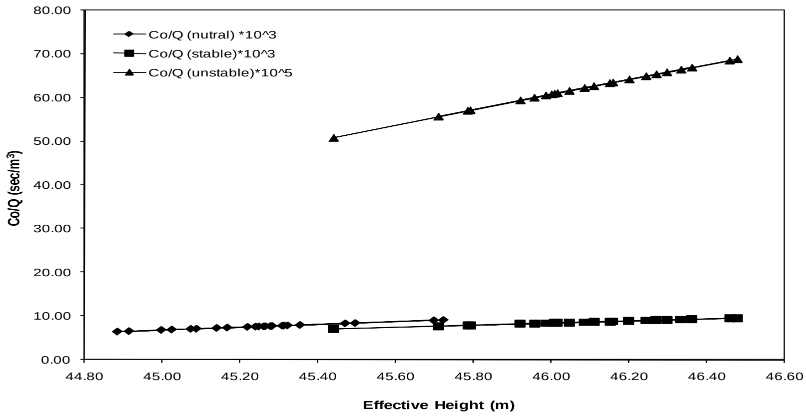

Fig.3. Variation of the the concentration at the plume axis over emision rate with effective height in neutral ,stable and unstable

Table 2. Observed wind speed, the plume rise, effective height and our proposed model, using power and using logarithmic Laws of the concentration at the axis of the plume at the reactor release over emission rate in neutral classes.

U (m/s)	Δh(m)	H(m)	Our proposed Model Co/Q *10^3 (sec/m^3)	Using power Law Co/Q*10^3 (sec/m^3) [2]
5.27	2.28	45.28	7.72	8.17
5.31	2.26	45.26	7.66	8.11
5.34	2.25	45.25	7.62	8.07
6.37	1.88	44.88	6.47	8.83
5.17	2.32	45.32	7.86	8.32
4.45	2.7	45.7	9.02	9.57
5.1	2.35	45.35	7.96	8.43
4.81	2.49	45.49	8.4	8.9
4.81	2.26	45.26	8.4	8.13
4.86	2.47	45.47	8.32	8.8
5.36	2.24	45.24	7.6	8.04
5.19	2.31	45.31	7.83	8.29
4.41	2.22	45.22	9.09	7.97
5.54	2.17	45.17	7.37	7.79
5.2	2.31	45.31	7.81	7.27
5.61	2.14	45.14	7.28	7.7
5.79	2.07	45.07	7.07	7.48
6.27	1.91	44.91	6.56	6.94
5.93	2.02	45.02	6.92	7.31
6.01	2	45	6.83	7.22
5.41	2.22	45.22	7.53	7.97
5.75	2.09	45.09	7.12	7.53

Figure 3 shows the relation between the effective height and Co/Q in different stability conditions.

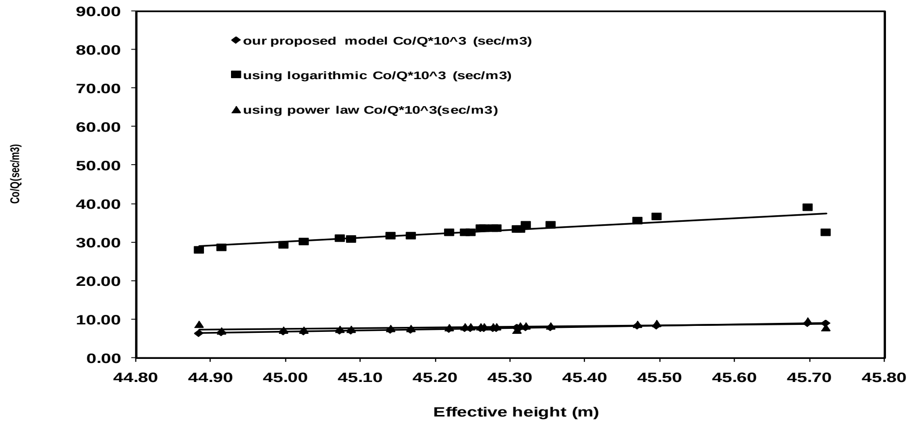

Fig.4..show the relation between the effective height and Co/Q using our proposed model and power,logarithmic laws in neutral case

Figure 4 and Table 2 Show the relation between Effective height and C_o/Q and comparison between our proposed model and using power and logarithmic laws, we find that there is a agree between our proposed model and using power law in neutral case but in the case of the logarithmic law the curve take the same biased.

Figure 5 and Table 3 Show the relation between Effective height and Co/Q between our proposed model, and using power, logarithmic laws, we find that there is a agree between our proposed model and using power law in stable case.

Table 3. Observed wind speed, the plume rise, effective height and our proposed model ,using power law and Using logarithmic Law of the concentration at the axis of the plume at the reactor release over emission rate in stable classes L=55m and β=5.

U(m/s)	Δh(m)	H(m)	Our proposed Model $Co/Q*10^3$ (sec/m3)	Using Power law $Co/Q*10^3$ (sec/m^3 Essa(2005)	Using logarithmic law $Co/Q*10^3$ (sec/m^3) Essa(2011)
4.43	2.28	45.28	7.66	8.7	17.99
3.81	2.26	45.26	8.75	9.93	20.97
4	2.25	45.25	8.39	9.5	19.64
4.92	1.88	44.88	6.97	7.87	16.62
3.7	2.32	45.32	8.98	10.19	20.9
3.57	2.7	45.7	9.27	10.52	21.61
3.64	2.35	45.35	9.11	10.34	21.66
3.45	2.49	45.49	9.55	10.85	22.38
3.6	2.26	45.26	9.2	10.45	21.63
3.8	2.47	45.47	8.77	9.95	20.97
3.99	2.24	45.24	8.4	9.52	19.63
3.89	2.31	45.31	8.59	9.75	20.27

3.75	2.22	45.22	8.88	10.07	20.94
3.98	2.17	45.17	8.42	9.54	19.63
3.47	2.31	45.31	9.5	10.79	22.39
4.06	2.14	45.14	8.28	9.37	19.67
4.3	2.07	45.07	7.86	8.9	18.51
4.31	1.91	44.91	7.85	8.88	18.52
4.02	2.02	45.02	8.35	9.46	19.65
4.11	2	45	8.19	9.27	19.04
3.94	2.22	45.22	8.5	9.63	20.3
3.86	2.09	45.09	8.65	9.81	20.26
3.67	2.28	45.28	9.05	10.27	21.68

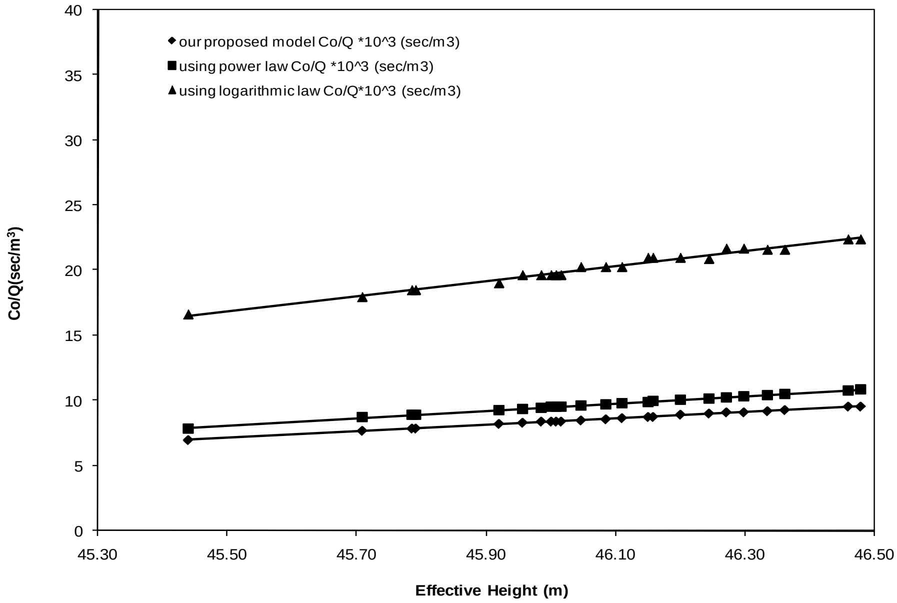

Fig.5. Show the relation between the effective height and C_o/Q using our proposed an power law ,logarithmic laws in stable case

Table 4. Observed wind speed, the plume rise, effective height and our proposed model ,using power law and Using logarithmic Law of the concentration at the axis of the plume at the reactor release over emission rate in unstable classes L= -2.5m and β=15

U(m/s)	Δh(m)	H(m)	Our proposed **M**odel Co/Q*10^6 (sec/m^3)	Using power Law Co/Q*10^6 (sec/m^3) Essa (2005)	Using logarithmic **L**aw Co/Q*10^6 (sec/m^3) Essa (2011)
4.43	2.28	45.28	55.75	9.68	73.97
3.81	2.26	45.26	63.43	11.13	76.15
4	2.25	45.25	60.86	10.65	75.8
4.92	1.88	44.88	50.87	8.78	72.22
3.7	2.32	45.32	65.01	11.44	76.6
3.57	2.7	45.7	66.99	11.82	77.64
3.64	2.35	45.35	65.91	11.61	77.13
3.45	2.49	45.49	68.93	12.19	78.07
3.6	2.26	45.26	66.53	11.73	77.07
3.8	2.47	45.47	63.57	11.16	76.74
3.99	2.24	45.24	60.99	10.67	75.79
3.89	2.31	45.31	62.32	10.1	76.26
3.75	2.22	45.22	64.28	11.3	76.67
3.98	2.17	45.17	61.12	10.69	75.78
3.47	2.31	45.31	68.6	12.13	77.48
4.06	2.14	45.14	60.09	10.24	75.29
4.3	2.07	45.07	57.2	9.96	74.4
4.31	1.91	44.91	57.08	9.94	74.41
4.02	2.02	45.02	60.6	10.6	75.82
4.11	2	45	59.46	10.38	75.34
3.94	2.22	45.22	61.65	10.79	75.73
3.86	2.09	45.09	62.73	11	76.22
3.67	2.28	45.28	65.46	11.52	77.17

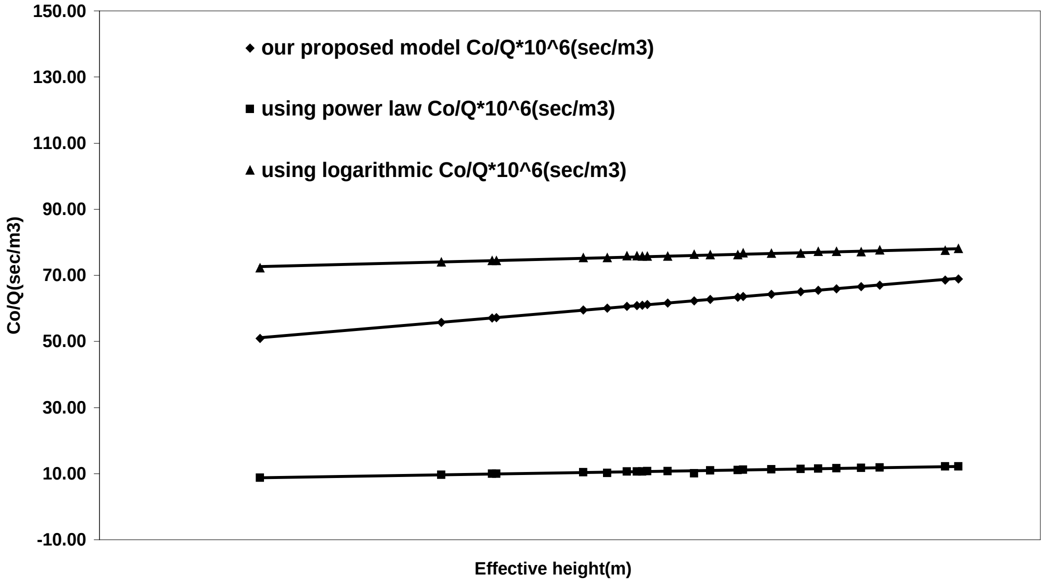

Fig.6. show the ralation between the effective height and Co/Q using our proposd model andusing power,logarithmic laws in unstable case

Figure 6 and Table 4 Show the relation between Effective height and Co/Q between our proposed models and using power, logarithmic laws, we find that there is agreement between our proposed model and using logarithmic law in unstable case.

5. Verification

The dosage ″D″ is the time integral of the concentration of a point source as follows:

$$D = \int_0^t C \, dt \tag{6}$$

where C is the concentration, e.g., g/m^3 or Bq/m^3.

For an instantaneous point source and assuming non-absorbing boundary, the mass balance condition is:

$$Q = \int_0^z \int_0^t U(z) C(z) \, dz \, dt = \text{Constant} \tag{7}$$

Using Eq. (6)

$$Q = \int_0^z U(z) D \, dz = \text{Constant} \tag{8}$$

where:-

Q is the total material discharge per unit second of the source g/s or Bq/s.

U is the wind speed m/s.

D is the dosage of material in g. s. m^{-3} or Bq. s. m^{-3}.

For a point source locate at height H1=27m (height of the source of the Research Reactor from the ground. For Iodine (I138), the height of the plume (H) is 31.29m, the total material discharge per unit second (Q) is 35Bq, the wind speed (u) is 2.8 m/s and the lapse rate ($\Delta T/\Delta Z$) (0C/100m) is 0.36. This is the case of stable, using Eq. (4); we get the concentration at the axis of the plume (C_0) equals 0.86 Bq/m^3. Then the concentration at the ground modifies to

$$C\,(ground) = 0.86\left(1-\frac{H_1}{H}\right) = 0.86\left(1-\frac{27}{31.29}\right) = 0.12Bq\,/\,m^3$$

The observed concentration at a distance x=300 m (with corresponds H=31.29m) was only 0.16 Bq/m^3.The calculated emission rate:

$$Q = \frac{(C_{obs.})(Q_{obs.})}{(C_{cal.})} = \frac{(0.16)(35)}{(0.12)} = 46.67\ \text{Bq}$$

By using Q (calculated), we find that the concentration at the ground "C" =0.16 Bq/m^3.

6- Summary and conclusions

The proposed model described the pollutant concentrations of a point source emitting pollutants into the atmosphere, the point source being located normal to the mean wind direction. The results of this study demonstrate: Wind shear, i.e. variation of wind with the height, is an important factor in turbulent diffusion problems using combination between logarithmic and power laws of wind speed. In our work, we estimated Co/Q for our proposed model and comparing with previous work using logarithmic and power laws in different stability conditions. We find that there is agreement between our proposed model and the previous model when we used power law in the neutral and stable conditions. Also there is agreement between our proposed model and the previous model when using logarithmic law in unstable stability. Also we calculate the concentration at the ground of the Iodine (I_{138}) which agrees with the observed concentration value after adjusted its source strength.

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