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Research Article

Stress and Stability analysis of viscoelastic rotating flow

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Abstract

Stress and Stability analysis of Viscoelastic rotating flow is studied in the narrow gap limit. The Galerkin projection method is used to derive dynamical System. Flow parameters are obtained using Mathematica Software. Stresses are computed in a wide range of the fluid elasticity effects. It is found that, all stress components decreased as fluid elasticity increased. Also the influence of fluid elasticity on the stability Taylor vortices is examined for the flow. It is observed that, the fluid elasticity tends to precipitate the onset of Taylor vortices. However, unlike the fluid elasticity, fluid retardation tends to delay the onset of Taylor vortices.

Keywords:

Stress ,
Viscoelastic,
fluid elasticity,
fluid retardation.

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1. Introduction

Fluids contained between two concentric cylinders rotating in the same and the opposite direction has been the subject of many experimental, analytical and numerical studies. This interest is motivated by the several industrial applications of this device such as journal bearings ,purification of industrial waste water and oil drilling. Taylor [1] conducted an experimental and theoretical investigation on the stability of a viscous flow in a small annular gap between two concentric cylinders with the inner one rotating. Diprima [2] found that the flow is relatively stable when the outer cylinder is rotating. H.-S. Dou et al. [3] proposed a method for calculating the energy loss distribution in the Taylor–Couette flow between concentric rotating cylinders. The principle and the detailed derivation for the calculation are given for a single cylinder rotation of either the inner or outer cylinder. They considered the cases of rotations in the same direction and the opposite direction .Benjamin [4] gave a qualitative description of the phenomenon of bifurcation and morphogenesis of the flow structure in the annular space with an aspect ratio of about 1 .The experimental results were presented for the location of bifurcation critical points and flow profiles .Researchers have dedicated many studies to Newtonian fluids despite, in practical applications the fluids drift away from the Newtonian idealized model. Thus, to take into account of this aspect more and more studies are devoted to non-Newtonian fluids. Although history shows sizeable literature on shear-thinning fluids [5], there appears to be a demand for analysis of viscoelastic fluids [6].The proposed model here is built upon models developed earlier for the flow of both Newtonian and non-Newtonian fluids in the gap between two concentric cylinders [7 and 8] can predict fluid response in practical range of shear rates. The Newtonian model [7] that leads to a well-known Lorenz system with the Prandtl number equal to unity cannot predict the destabilization of the Taylor vortices. The recent study of Ashrafi and Karimi-Haghighi [8] looked at the effect of the gap width on the stability of the non-Newtonian Taylor-Couette flow. The study used a spectral method with a low number of truncation modes with a more crude expression for the viscosity. Expectedly, the effect of gap has resulted in a more complicated dynamical system, of which the solution is provided in the study.

In this study, Stress and Stability analysis of Viscoelastic rotating flow is studied in the narrow gap limit. The Galerkin projection method is used to derive dynamical System. Flow parameters are obtained using Mathematica Software. Stresses are computed in a wide range of the fluid elasticity effects. Also, the influence of fluid elasticity and fluid retardation on the stability Taylor vortices is examined for the flow.

2. PROBLEM FORMULATION

Consider an incompressible viscoelastic fluid of density ρ and viscosity μ. In this study, we shall consider fluids that can be reasonably represented by a single relaxation time and constant viscosity. The fluid is assumed confined between two infinite and concentric cylinders of inner and outer R_1 and R_2, respectively. The inner cylinder is assumed to rotate at a constant angular velocity, Ω, while the outer cylinder is at rest. Here, the flow is governed by the following conservation of mass and linear momentum equations for an incompressible fluid:

$$\nabla . U = 0 \tag{1}$$

$$\rho\left(\frac{\partial U}{\partial T} + U.\nabla U\right) = -\nabla.\Sigma \tag{2}$$

Where ∇ is the gradient operator, T is the time, U is the velocity vector, and Σ, is the stress tensor.

$$\Sigma = PI - \mu\Gamma + T$$

$$T + \lambda_1 \check{T} = -\mu\left(\Gamma + \lambda_2 \check{\Gamma}\right) \tag{3}$$

$$\check{T} \equiv \frac{\partial T}{\partial T} + U.\nabla - (\nabla U)^t.T - T.\nabla U$$

Upon Galerkin projection and extensive manipulations, we finally arrive at the following dynamical system:

$$\begin{aligned}
\dot{U} &= V + X - aRvU \\
\dot{V} &= -UW + rU + Y - aRvV \\
\dot{W} &= UV + b(Z - aRvW) \\
\dot{X} &= -\delta(X + aU) \\
\dot{Y} &= -bUZ - \varphi WX - rX - \delta(Y + aV) \\
\dot{Z} &= \beta(UY - \varphi VX) - \delta(Z + aW)
\end{aligned} \tag{4}$$

Where:

$$\begin{aligned}
u &= \frac{\pi\tau}{\sqrt{2}} u_x^{11}, \\
v &= \frac{\pi}{\sqrt{2}} r\, u_y^{11}, \\
w &= -\pi\, r u_y^{20} \\
x &= \frac{\pi k^2 \tau^3}{\sqrt{2}} \left(\pi(\tau_{xx}^{11} - \tau_{zz}^{11}) + \frac{\pi^2 - k^2}{k} \tau_{xz}^{11} \right) \\
y &= \frac{\pi\tau}{\sqrt{2}} r\left(\pi\tau_{xy}^{11} - k\tau_{yz}^{11}\right) \\
z &= -\frac{r\tau_{xy}^{20}}{2}
\end{aligned} \tag{5}$$

The following parameters were introduced:

$$\begin{aligned}
&r = k^2\tau^3 Ta\,, \qquad b = 4\pi^2\tau\,, \qquad \varphi = (3\pi^2 - k^2)\tau\,, \\
&\tau = \frac{1}{\pi^2 + k^2}\,, \qquad \delta = \frac{\tau}{E}\,, \qquad \beta = \frac{1}{b}
\end{aligned} \tag{6}$$

Here, *Ta* is the Taylor number, which is given in terms of Reynolds number, *Re*, and the ratio of gap width to the inner cylinder radius, ε, as following**:**

$$Re = \frac{dR_i\Omega}{v}\,, \qquad \varepsilon = \frac{d}{R_i}\,, \qquad Ta = \varepsilon Re^2 \tag{7}$$

Other parameters, *E* and *Rv*, are a measure of fluid elasticity and a measure of fluid retardation respectively and are presented in Equations (8).

$$E = \frac{\lambda_1\, v}{d^2}\,, \qquad Rv = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{\mu_s}{\mu_p}\,, \qquad a = \frac{\mu_p}{\mu} = \frac{1}{Rv + 1} \tag{8}$$

3. Stability analysis

The stability analysis of the flow is carried out around the CCF first. The analysis is based on the linearization of Equations (4). The critical Taylor number corresponding to the emergence of Taylor vortices $\mathrm{Ta_c}$ is given by:

$$Ta_c = \left(\frac{(k^2+\pi^2)^3}{k^2}\right)\left(\frac{\tau}{\tau+aE}\right) \tag{9}$$

The marginal stability curves are depicted in Fig. 1, where the critical Taylor number is plotted against the wave number. The figure shows that the critical Taylor number becomes lower as fluid elasticity increases. In other words, it is found that fluid elasticity tends to precipitate the stability of the CCF. Also, there is a slight shift in the value of k, toward the right as E increases, thus making it more difficult Taylor vortices to be observed.

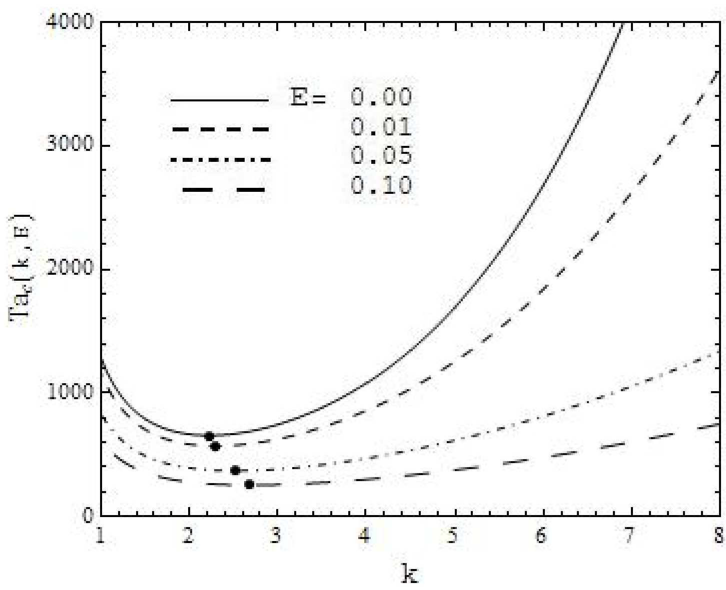

Fig. 1: Marginal stability curves for various E values and $Rv = 0$.

The influence of fluid retardation on the critical Taylor number is depicted from Fig. 2 for a fluid with $E = 0.1$. The figure displays the stability curves for various values of the viscosity ratio Rv. As Rv increases, Ta, tends to increase. Thus, fluid retardation tends to delay the onset of Taylor vortices.

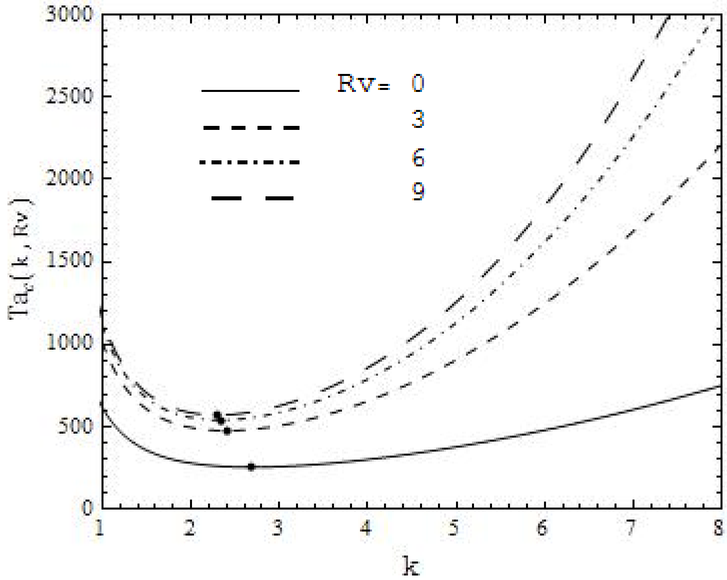

Fig. 2: Marginal stability curves for various Rv values and $E = 0.1$.

4. Stress analysis

A major objective of the current study is to investigate variation of developed stresses. The stresses are evaluated and density plot of the most important component of the stress tensor i.e. T_{xx} and T_{xz} are depicted in Figs.3 to Figs.7. It can be seen from the figures that, all stress components decreased as fluid elasticity increased.

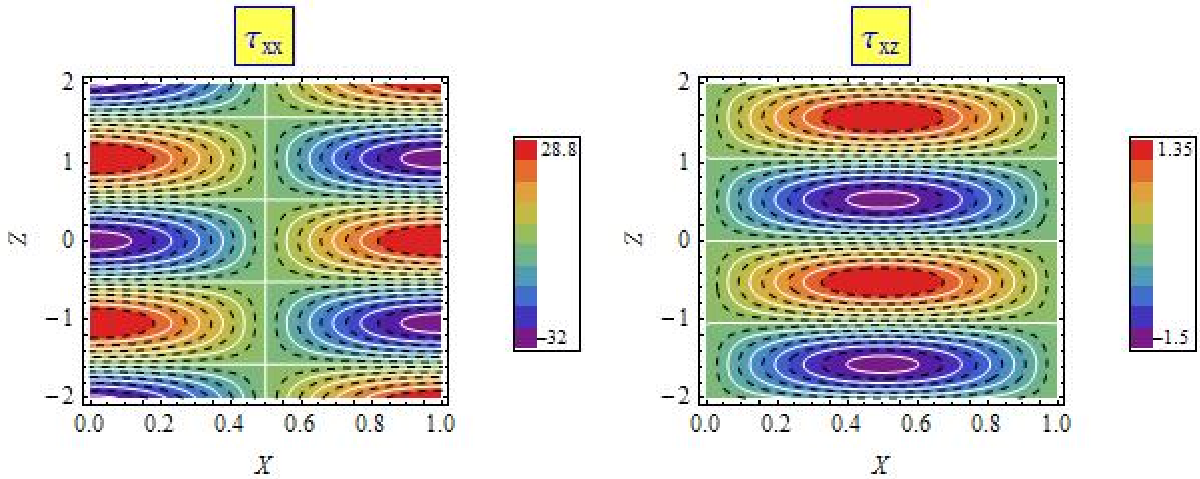

Fig. 3: Stress maps in the x-z plane at $E = 0.04, Rv = 1.0, k = 3$.

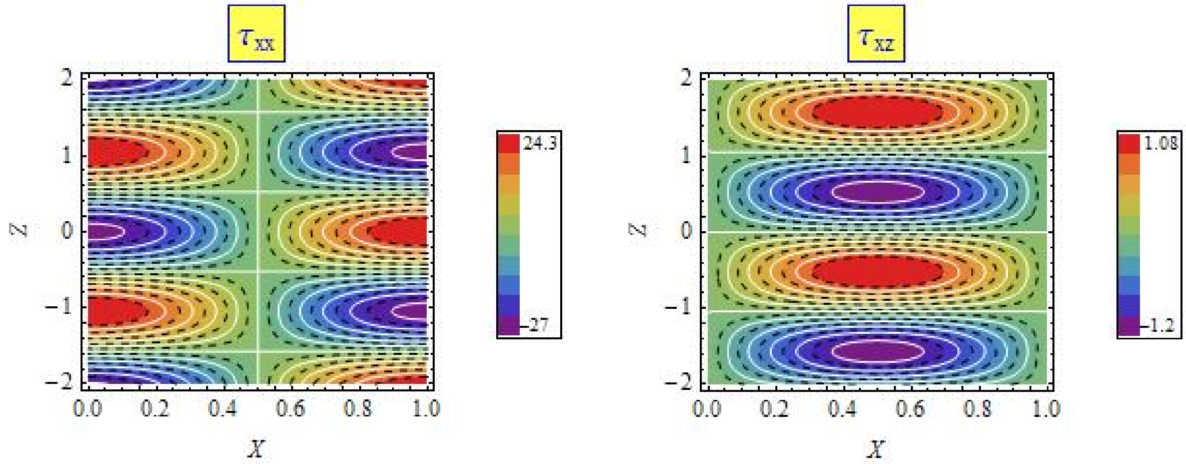

Fig. 4: Stress maps in the x-z plane at $E = 0.08, Rv = 1.0, k = 3$.

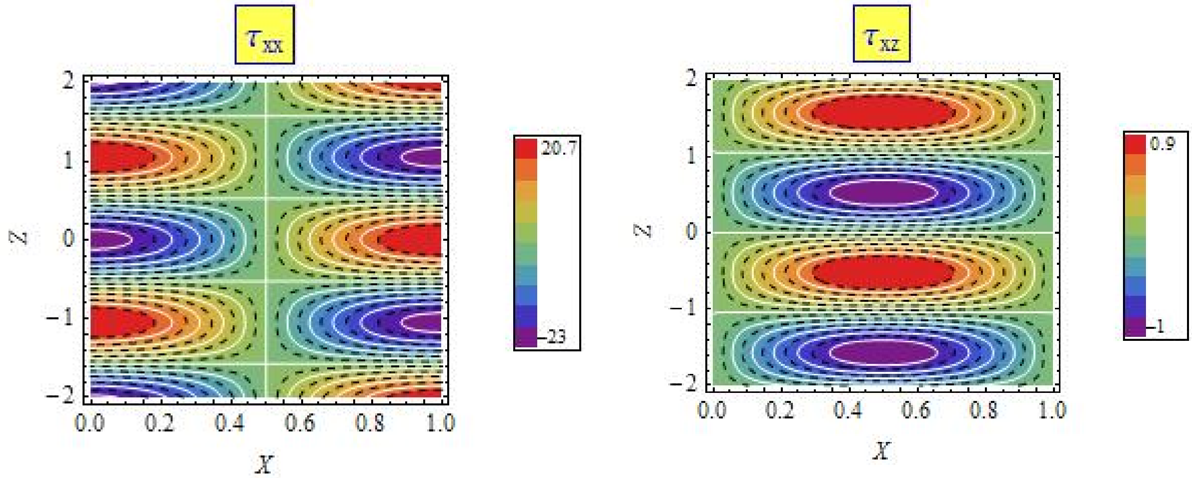

Fig. 5: Stress maps in the x-z plane at $E = 0.12, Rv = 1.0, k = 3$.

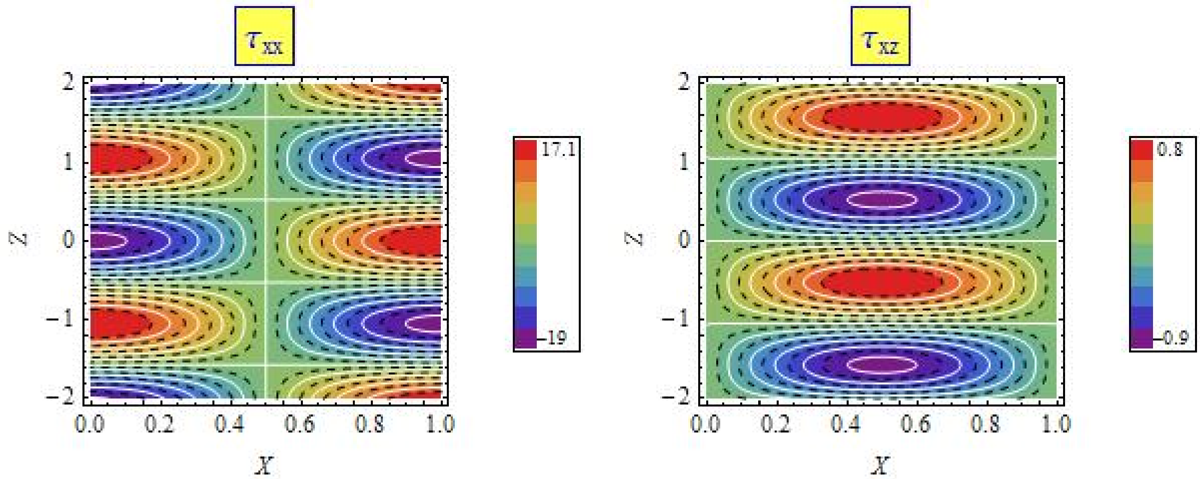

Fig. 6: Stress maps in the x-z plane at $E = 0.2, Rv = 1.0, k = 3$.

4. Conclusions

A dynamical system has been developed for the flow of viscoelastic fluids between two concentric cylinders using the Galerkin projection method. Stresses are computed in a wide range of the fluid elasticity effects. Also, the influence of fluid elasticity and fluid retardation on the stability Taylor vortices is examined for the flow. It is found that, all stress components decreased as fluid elasticity increased. Also, it is observed that, the fluid elasticity tends to precipitate the onset of Taylor vortices. However, unlike the fluid elasticity, fluid retardation tends to delay the onset of Taylor vortices.

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