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Research Article

Mathematical Simulation of Instability Phenomena as a Compositional Model during Oil Recovery Process

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A b s t r a c t

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Mathematical models of petroleum reservoirs have been utilized since the late 1800s. A mathematical model consists of a set of equations that describe the flow of fluids in a petroleum reservoir. In this paper the applicability and effectiveness of a novel iterative method as a semi-analytical method is shown for solution of partial differential equation that model porous medium. This nonlinear partial differential equation describing instability phenomenon of two immiscible fluids (water and oil) flow through homogeneous porous cylindrical medium with impervious bounding surfaces on three sides of an oil reservoir.
This method applied to a porous medium equation arising in instability phenomena in double phase flow through porous media. Furthermore, this technique is utilized to find closed-form solutions for the problem under consideration with appropriate initial/boundary conditions.

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1. Introduction

For increasing the recovery of oil that contained in a porous matrix of an oil formatted region in an oil reservoir, the oil must be displaced by a fluid with lesser viscosity. This fluid can be water. The protuberances occur which shoot thorough the porous medium at relatively big speed Instead of regular displacement of the whole front. These protuberances are called fingers. This phenomenon is called instability. The statistical behaviour of instability was analysed by Scheidegger[1] in a displacement process inside a homogeneous porous medium with capillary pressure and pressure-dependent phase densities. This problem has great importance for oil production in petroleum technology. Movement of oil from a porous during secondary oil recovery the fractional flow formulation employs the saturation of one of the phases and a global pressure as independent variables. The fractional flow approach treats the two-phase flow problem as a total fluid flow of a single mixed fluid and then describes the individual phases as fractions of the total flow. This approximate leads to a less strong coupling between the two coupled equations: the global pressure equation and the saturation equation.

The understanding and prediction of multiphase flow through porous media is important in different areas of research and industry. In the fields of Petroleum engineering it is necessary to model multiphase and multicomponent flow for production of hydrocarbons from petroleum reservoirs.it is one of the important problems of petroleum technology and water hydrology in the oil-water movement in a porous medium Scheidegger[1] Buckley and Leveret obtained the motion of two immiscible fluids in a homogenous porous medium without considering capillary pressure. The oil and water form two immiscible liquid phases and in this paper water assumed wetting phase.

We focus our attention on the study of immiscible compressible two phase flow in porous media taking into account gravity, capillary effects and heterogeneity.

Some researchers studied two immiscible incompressible two-phase flows by using the feature of Global pressure; for instance Antontsev et al. [2], Chavent and Jaffr´e[3], Chen et al. [4].The standard continuum models can indeed be used for describing flow in heterogeneous porous media demonstrated by chaouche et al. [5]. Both experiments and calculations based on pore network simulations show a good qualitative agreement with the predictions based on the continuum approach. The influence of capillarity on flow through fractal permeability fields is investigated numerically by batenburg, et al. [6]

Also more researchers such as chavent and Jaffr´e[3], Galusinski and Saad[7]studied two compressible phase's flows with small capillary pressure by assuming that the densities are depend on the global pressure. Homogenization theory can be applied successfully under the condition of separation of scales see bensoussan et al. [8] and bourgeat[9]. This approach fails if heterogeneity occurs at all length scales, or in the case of capillary dominated drainage, as this is a percolation process.

The goal of this paper is to solve the nonlinear partial differential equation that describe instability phenomenon of water and oil flow through homogeneous porous cylindrical medium with impervious bounding surfaces on three sides of an oil reservoir and obtain the saturation of water $s_w(X,T)$ as a function of distance and time in displaced oil zone.

This nonlinear partial differential equation is solved by Adomian's decomposition scheme for the case of instability phenomena RamakantaMeher et al. [10].The iterative method is used before for solution Korteweg-de Vries equations by FarshadEhsani, et al. [11] .In this paper this equation is solved by an iterative method and compared the results with the results of RamakantaMeher et al. [10]

2. Symbols

k = penetrance of homogenous medium

k_w = the relative penetrance of water (as a function of s_w)

k_o = the relative penetrance of oil (as a function of s_o)

s_w = saturation of water

s_o = saturation of oil

μ_w =kinematic viscosity of water phase in homogenous porous media

μ_o =kinematic viscosity of oil phase in homogenous porous media

Φ =porosity of the medium

p_c =capillary pressure

3. Basic Idea of Iterative Method

To illustrate the basic concepts of iterative method, considering the following partial differential equation

$$\mathrm{A}(\mathrm{u}(\mathrm{x,t}))-\mathrm{f}(\mathrm{x,t})=0 \quad r\in\Omega \quad (1)$$

Subject to boundary condition

$$\mathrm{B}(\mathrm{u},\partial \mathrm{u}/\partial \mathrm{n}) = 0 \quad x\in\Gamma \qquad (2)$$

Where A is a general differential operator, B a boundary operator $f(x)$ is a known analytical function, Γ is the boundary of domain Ω and $\frac{\partial u}{\partial n}$ denotes differentiation along the normal drawn outwards from Ω. The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Eq. (1) therefore can be rewritten as follows

$$\mathrm{L}(\mathrm{u}(\mathrm{x,t}))+\mathrm{N}(\mathrm{u}(\mathrm{x,t}))-\mathrm{f}(\mathrm{x,t})=0 \qquad (3)$$

In this method by the elimination of non-linear term, the initial guess is obtained as follows

$$\mathrm{L}(\mathrm{u}_0(\mathrm{x,t}))=\mathrm{f}(\mathrm{x,t}) \quad \mathrm{B}(\mathrm{u}_0,\partial \mathrm{u}_0/\partial \mathrm{n}) = 0 \quad x\in\Gamma \qquad (4)$$

Then by replacing the $\mathrm{u}_n(\mathrm{x,t})$ and $\mathrm{u}_{n+1}(\mathrm{x,t})$ respectively in the non-linear and linear term of PDE, next iterations obtained as Eq. (5)

$$\mathrm{L}(\mathrm{u}_{n+1}(\mathrm{x,t}))=-\mathrm{N}(\mathrm{u}_n(\mathrm{x,t}))+\mathrm{f}(\mathrm{x,t}) \qquad (5)$$

In fact each $u_i(x,t)$ is separately the solution of problem. The iterative method is easy to use and each solution is a betterment of the previous iteration, Continuing this manner, until could be obtained $u_n(x,t)$ which is in the good approximate with the exact solution.

4. Analysis

Darcy's law for each phase is written in the usual form

$$v_\alpha = -\frac{1}{\mu_\alpha} k_\alpha \left(\nabla p_\alpha - \rho_\alpha \wp \nabla z\right) \qquad \alpha = w, o, g \tag{6}$$

Where k_α , p_α and μ_α are the effective permeability, pressure, and viscosity for phase α . Since the simultaneous flow of two fluids causes each to interfere with the other, the effective permeability's are not greater than the absolute permeability k of the porous medium. The relative permeability's $k_{r\alpha}$ are widely used in reservoir simulation

$$k_\alpha = k_{r\alpha} k \;\; \alpha = w,o \tag{7}$$

The function $k_{r\alpha}$ indicates the tendency of phase α to wet the porous medium.

The seepage velocity of water (Vw) and oil (Vo) written as follow

$$v_o = -\frac{k_o k}{\mu_o} \times \frac{\partial p_o}{\partial x} \tag{8}$$

$$v_w = -\frac{k_w k}{\mu_w} \frac{\partial p_w}{\partial x} \tag{9}$$

Total mass of each component must be conserved

For water phase:

$$\frac{\partial\left(\varphi \rho_w s_w\right)}{\partial t} = -\nabla.\left(\rho_w u_w\right) + q_w \tag{10}$$

When q_w represent a finite number of point sources or sinks.

By assuming constant density and neglecting q_w

$$\varphi \frac{\partial s_w}{\partial t} + \frac{\partial v_w}{\partial x} = 0 \tag{11}$$

For oil phase:

$$\frac{\partial(\varphi\rho_o s_o)}{\partial t} = -\nabla.(\rho_o u_o) + q_o \qquad (12)$$

When q_o represent a finite number of point sources or sinks.

Then by assuming the constant density and neglecting q_o for the oil phase

$$\varphi\frac{\partial s_o}{\partial t} + \frac{\partial v_o}{\partial x} = 0 \qquad (13)$$

The fact that the two phases jointly fill the void space is given by the equation

$$s_o + s_w = 1 \qquad (14)$$

Where s_w and s_o are the saturations of the wetting and non-wetting phases, respectively. Also, due to the curvature and surface tension of the interface between the two phases, the Pressure in the wetting fluid is less than that in the non-wetting fluid. Finally, the phase pressure is related by capillary pressures

$$p_c = p_o - p_w \qquad (15)$$

By substituting equations (8)-(15) in each other's

$$\varphi\frac{\partial s_w}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}\left[k\frac{k_w}{\mu_w}\frac{dp_c}{ds_w}\frac{\partial s_w}{\partial x}\right] = 0 \qquad (16)$$

Scheidegger[1] defined k_w as a function of s_w

Standard form of Capillary pressure is defined by verma[12] as follow

$$p_c = \beta\left(c_0 + s_w^{-2}\right), \quad \beta \text{ And } c_0 \text{ are constants} \qquad (17)$$

When

$$k_w = s_w \qquad (18)$$

By substituting equations (17), (18) in the equation (16)

$$\frac{\partial s_w}{\partial t} = \frac{k\beta}{\mu_w\varphi}\left(\frac{\partial s_w}{\partial x}\times\frac{\partial}{\partial x}\left(\frac{1}{s_w^2}\right) + \frac{\partial^2 s_w}{\partial x^2}\times\left(\frac{1}{s_w^2}\right)\right) \qquad (19)$$

The nonlinear partial differential equation (19) describe the instability phenomenon of two immiscible fluids flow in the homogeneous porous cylindrical medium with impervious bounding surfaces on three sides of an oil reservoir.

The following dimensionless parameters are used

$$X = \frac{x}{L} \quad , \quad Z = \frac{k\beta}{\phi \mu_w L^2 t} \tag{20}$$

$$\frac{\partial s_w}{\partial z} = \frac{\partial}{\partial X}\left(\frac{1}{s_w^2}\frac{\partial s_w}{\partial X}\right) \tag{21}$$

The initial and boundary equation for equation (21) are defined by verma[12] as follows

$$s_w(X,0) = s_{w,0}(X) = \left(X^2+1\right)^{\frac{-1}{2}} \tag{22}$$

$$S_w(0,z) = g_1(z), \qquad S_w(L,z) = g_2(z) \tag{23}$$

For solution of Eq (21) with initial condition (22) and boundary conditions (23) it is used of iterative method

$$\mathrm{L}\left(\mathrm{u}(\mathrm{x},z)\right) = \mathrm{u}_Z \tag{24}$$

$$\mathrm{N}\left(\mathrm{u}(\mathrm{x},z)\right) = -\left(\frac{\partial u}{\partial X}\times\frac{\partial}{\partial X}\left(\frac{1}{u^2}\right)+\frac{\partial^2 u}{\partial X^2}\times\left(\frac{1}{u^2}\right)\right) \tag{25}$$

$$\mathrm{f}(\mathrm{x},z) = 0 \tag{26}$$

$$\mathrm{L}\left(\mathrm{u}(\mathrm{x},z)\right)+\mathrm{N}\left(\mathrm{u}(\mathrm{x},z)\right)-\mathrm{f}(\mathrm{x},z) = 0 \tag{27}$$

$$\mathrm{u}_0(\mathrm{x},0) = \mathrm{u}_0 = \left(X^2+1\right)^{\frac{-1}{2}}, \mathrm{u}_{\mathrm{n}+1}(\mathrm{x},0) = \left(X^2+1\right)^{\frac{-1}{2}} \tag{28}$$

$$\left(\frac{\partial \mathrm{u}_{\mathrm{n}+1}}{\partial z}\right) = -\left(\frac{\partial u_n}{\partial X}\times\frac{\partial}{\partial X}\left(\frac{1}{u_n^{\ 2}}\right)+\frac{\partial^2 u_n}{\partial X^2}\times\left(\frac{1}{u_n^{\ 2}}\right)\right)+0 \tag{29}$$

By assuming above initial and boundary conditions a solution for equation (29) by iterative method is as follows

$$u_0 = \left(X^2 + 1\right)^{\frac{-1}{2}} \quad (30)$$

$$u_1 = t \times \left(\left(\left(\frac{x^2}{\left(1+x^2\right)^{\frac{3}{2}}} \right) - \left(\frac{1}{\sqrt{1+x^2}} \right) \right) + \left(\left(x \wedge 2 + 1\right)^{\frac{-1}{2}} \right) \right) \quad (31)$$

Each of above terms is the solution of equation (29) but must be choosing the best answer by comparing u_{n-1} and u_n until that $\left|u_n - u_{n-1}\right| \le e$; e=computational accurate

It can be written more iteration for better solution; although next iteration is better that previous iteration. Everyone can probe this statement by comparing each u_n with u_{n-1} .In this example after just two iterations. So u_1 is the solution of equation (21). It could be obtaining the semi exact analytical solution with a good approximate with Adomian's decomposition method and generalized conditional symmetry method RamakantaMeher et al. [10]

5. Results

In this paper, iterative method has been successfully applied to find the solution of the saturation of water $S_w\left(X,T\right)$ as a function of distance and time in displaced oil zone. Then the solution is compared with the $S_w\left(X,T\right)$ as an instability phenomena that obtained from Adomian's decomposition method and Generalized conditional symmetry method RamakantaMeher et al.[10] This comparing shown the Iterative method has the best accuracy against of past methods and proved the convergence of the iterative method just after obtaining two sentences. The best advantage of iterative method is it's simplify and accurate and saving time. The results of the iterative method are in approximately agreement with past results. The iterative method is capable to solve other partial differential equations and it can be used instead of other methods for finding semi-analytical solution of PDE. Table (1) and (2) and (3) show the Difference between iterative method and Adomian's decomposition method and Generalized conditional symmetry method in solving of the instability phenomenon of two immiscible fluids flow in the homogeneous porous cylindrical medium for different values of x and t .In the figures (1) and (2) two-dimensional plot is shown for the comparison of each solution of $s_w(X,T)$ that it is obtained from iterative method and ADM and Generalized conditional symmetry method for different values of x and t.

Table1: Saturation versus time keeping distance fixed: Adomian's decomposition method RamakantaMeher et al. [10]

X/T	0.1	0.2	0.3	0.4	0.5
0.1	0.901153	0.815959	0.738585	0.668161	0.603819
0.2	0.890376	0.807984	0.732875	0.664522	0.602394
0.3	0.873241	0.795169	0.723512	0.658171	0.59905
0.4	0.850831	0.778173	0.7108	0.649006	0.593088
0.5	0.8244	0.757806	0.69522	0.637214	0584359
0.6	0.795209	0.734933	0.677375	0.623243	0.573248
0.7	0.764408	0.710386	0.657894	0.60766	0.560411
0.8	0.732959	0.684903	0.637365	0.591013	0546511
0.9	0.701618	0.659097	0.616291	0.573761	0.532069
1	0.670941	0.63345	0.595075	0.556257	0.51744

Table 2: Generalized conditional symmetry method RamakantaMeher et al. [10]

X/T	0.1	0.2	0.3	0.4	0.5
0.1	0.901156	0.816	0.738794	0.668819	0.605418
0.2	0.890375	0.80797	0.732818	0.664376	0.602117
0.3	0.873237	0.795098	0.723174	0.657164	0.596732
0.4	0.850824	0.778069	0.710289	0.64745	0.589432
0.5	0.824393	0.757701	0.694693	0.635572	0.580427
0.6	0.795204	0.734852	0.676957	0.621905	0.569962
0.7	0.764406	0.710342	0.65765	0.606837	0.558295
0.8	0.732959	0.684895	0.637302	0.59074	0.545684
0.9	0.70162	0.659119	0.616378	0.57396	0.532376
1	0.670944	0.633492	0.595268	0.556799	0.518596

Table 3: present research by iterative method

X/T	0.1	0.2	0.3	0.4	0.5
0.1	0.896519	0.798	0.699482	0.600963	0.502445
0.2	0.886294	0.792007	0.697721	0.603434	0.509148
0.3	0.869952	0.782078	0.694204	0.60633	0.518456
0.4	0.848436	0.768395	0.688353	0.608312	0.528271
0.5	0.822873	0.751319	0.679765	0.60821	0.536656
0.6	0.794442	0.731391	0.66834	0.605289	0.542238
0.7	0.76425	0.709268	0.654286	0.599304	0.544322
0.8	0.733255	0.685641	0.638027	0.590413	0.542799
0.9	0.702228	0.661162	0.620096	0.57903	0.537964
1	0.671751	0.636396	0.601041	0.565685	0.53033

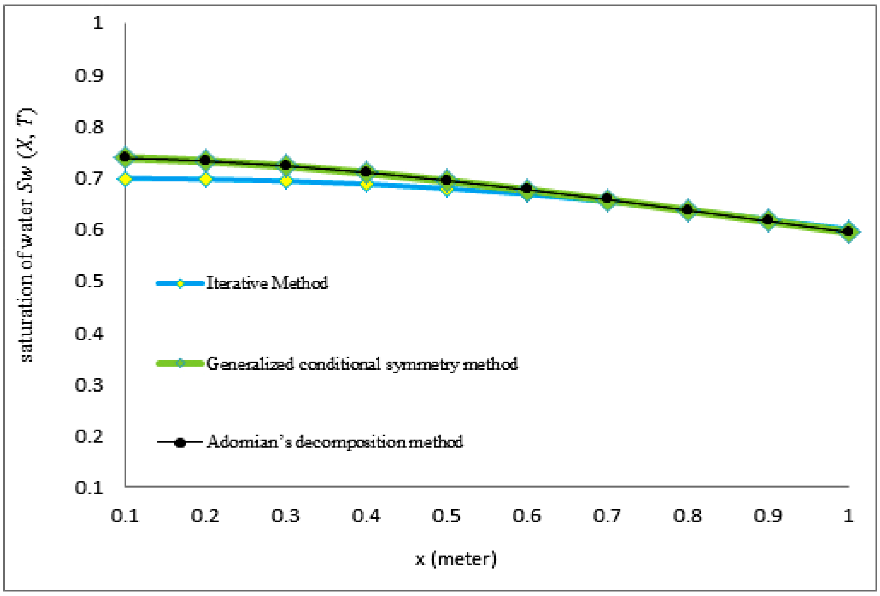

Fig.1: The comparison of Iterative Method, ADM and Generalized conditional symmetry method for obtaining $S_w(X,T)$ for difference values of X and $T = 0.3$

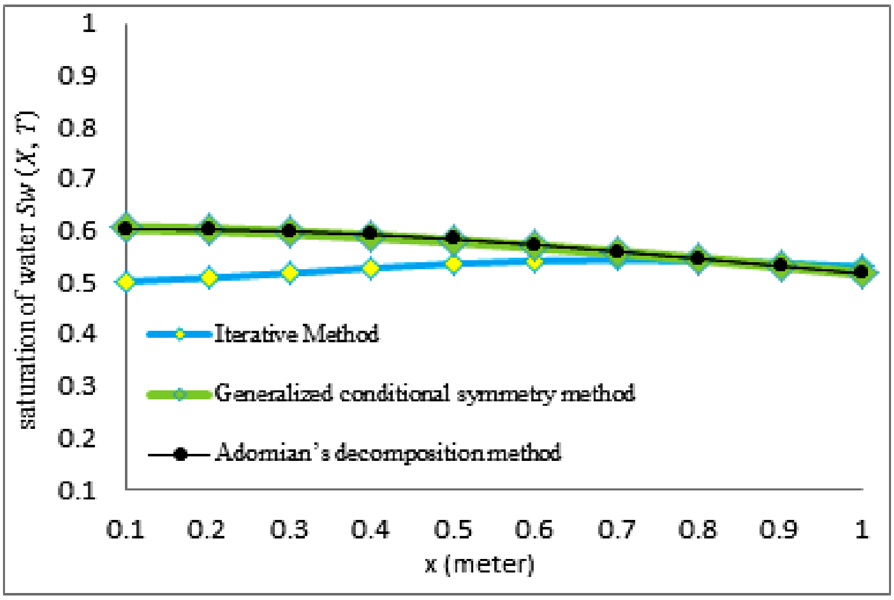

Fig.2: The comparison of Iterative Method, ADM and Generalized conditional symmetry method for obtaining $S_w(X,T)$ for difference values of X and $T = 0.5$

6. Conclusions

The following conclusions can be drawn from this research:

i. The Iterative method has the best accuracy against of past methods and proved the convergence of the iterative method just after obtaining two sentences.
ii. The best advantage of iterative method is it's simplify and accurate and saving time.

iii. It could be obtaining the semi exact analytical solution with a good approximate with Adomian's decomposition method and generalized conditional symmetry method RamakantaMeher.

iv. Nonlinear partial differential equation is solved by Adomian's decomposition scheme for the case of instability phenomena RamakantaMeher and it showed accurate results.

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