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Seismic Wave Propagation and Imaging in Anisotropic Media: A Review

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Abstract

The concept of seismic anisotropy is applied extensively in seismic modelling, imaging and inversion. Alignment of mineral grains, clay platelets, layering, and fractures all contribute to the observed anisotropy in the subsurface. Common problems caused by ignoring anisotropy in seismic imaging include mis-tie in time-to-depth conversion, failure to preserve dipping energy during dip-moveout (DMO) correction, and mispositioning of migrated dipping events. Proper treatment of anisotropy during the processing of seismic data not only helps to avoid distortions in reservoir imaging, but also provides estimates of the anisotropy parameters, which carry valuable information about lithology and fracture networks. To consider the influences of seismic anisotropy in imaging, an anisotropic wave equation needs to be employed. Depending on the type of anisotropic model, various wave equations are introduced, which can be used for both seismic modeling and imaging. In this article, developed algorithms for employing anisotropy effect in seismic modeling and imaging are presented with emphasizing on acoustic approximations.

1. Introduction

The Earth's subsurface is well known for its anisotropic properties (Thomsen, 1986). Thin layering, alignment of mineral grains, clay platelets, and fractures all contribute to the observed anisotropy in the Earth. The implementation of anisotropy in exploration became more essential when seismic data were acquired with wider offsets and azimuths (Macbeth and Lynn, 2000). Anisotropy is defined as the variation of one or more properties of a medium with direction. In the case of seismic anisotropy, the velocity of seismic waves depends on the direction of propagation (Fig. 1).

Common problems caused by ignoring anisotropy in seismic imaging include lower image resolution, mis-tie in time-to- depth conversion, failure to preserve dipping energy during dip-moveout (DMO) correction, and mis-positioning of migrated dipping events (Alkhalifah, 1997; Moussavi Alashloo and Ghosh, 2018). Proper treatment of anisotropy during the processing of seismic data not only helps to avoid distortions in reservoir imaging, but also provides estimates of the anisotropy parameters, which carry valuable information about lithology and fracture networks.

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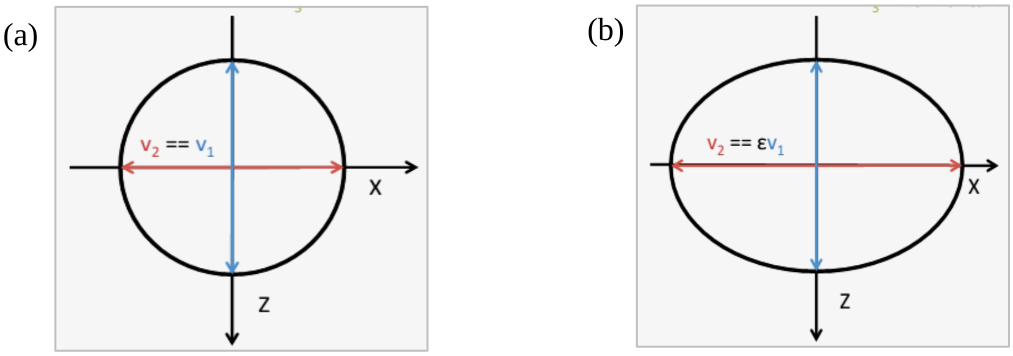

Figure 1. Difference of wave propagation in (a) isotropic and (b) anisotropic environment. Unlike the isotropic wavefront which is circle, the anisotropic wavefront is dependent on the angle of propagation. V1 is vertical velocity, V2 is horizontal velocity, and $\boldsymbol{\varepsilon}$ is anisotropic parameter (Krüger, 2012).

Currently, many seismic processing and inversion methods operate with anisotropic models, and there is little doubt that in the near future anisotropy will be treated as an inherent part of the velocity field (Tsvankin *et al.*, 2010). Techniques, which have developed in the last two decades to estimate the fracture parameters, utilize seismic data, mainly using VSP and surface seismic data. These techniques can be grouped into three categories: velocity analysis and imaging, S-wave splitting and mode-converted PS-waves, and AVO analysis (Wild, 2011; Yan and Han, 2020).

Multicomponent data, nowadays, is used commonly to characterize anisotropic media (Bansal and Sen, 2008; Wuestefeld *et al.*, 2011; Li and Mallick, 2014). Numerous researches were conducted by applying shear-wave splitting as one of the most robust indicators of seismic anisotropy and considered to be one of the most successful ways for detecting and characterizing fractures (Al-Harrasi *et al.*, 2011; Vasconcelos and Grechka, 2007; Haacke *et al.*, 2009; Chen *et al.*, 2017; Hoseini *et al.*, 2012). Travel-time and amplitude differences between the fast and slow shear waves, as well as their NMO ellipses, can help estimate fracture orientation, density and, in some cases, make inferences about fluid saturation (Kendall *et al.*, 2007). However, the majority of multicomponent offshore surveys are acquired without shear-wave sources, so the reflected wavefield is largely composed of compressional waves and mode-converted PS-waves (Tsvankin *et al.*, 2010). Recently, there has been renewed interest in developing a more formal inversion approach to the PS-wave layer-stripping problem where the objective function is formulated in terms of the PS-wave polarization azimuth and the traveltime difference between the split PS-waves (Bale *et al.*, 2009). It is important to note that the moveout asymmetry of PS-waves helps to constrain the parameters of tilted TI media (Dewangan and Tsvankin, 2006) and characterize dipping fracture sets (Angerer *et al.*, 2002).

Analysis of amplitude variations with offset (AVO) contains valuable information about the local media properties on both sides of an interface. AVO analysis indicates considerably more promise, in particular for estimation of dominant fracture directions in naturally fractured reservoirs (Vasconcelos and Grechka, 2007).The azimuthally varying P-wave AVO response has been successfully used for estimating the dominant fracture orientation and, in some cases, mapping "sweet spots" of intense fracturing (Xu and Tsvankin, 2007). For instance, Hall and Kendall (2003) demonstrate that the direction of the minimum AVO gradient at Valhall field is well-aligned with faults inferred from coherency analysis.

2. P-wave anisotropic approximations

In P-wave (acoustic) imaging, isotropic imaging algorithms have been mostly developed for tilted transverse isotropy (TTI) and vertical transverse isotropy (VTI) media (Alkhalifah and Fomel, 2009; Koren *et al.*, 2010; Behera *et al.*, 2011). Transverse isotropy (TI), the simplest form of anisotropy, exists when thin bed sequences, perpendicular to the symmetry axis, are isotropic. A medium is called VTI where thin beds are horizontal, and TTI where layers are tilted due to tectonic activity (Figures 2a and 2b). VTI anisotropy commonly exists in sedimentary media where lithification ordinarily happens by the vertically oriented compaction pressure. Strong tectonic stress can also create cracks and fractures in thin bed interfaces, which results in an azimuthal anisotropy (Lynn *et al.*, 2011; Fowler and King, 2011; Cyz and Malinowski, 2018). In this condition, the transverse isotropic theory cannot justify the discrepancy of residual moveouts among common image gathers (CIGs) of different azimuths. Orthorhombic, as a more inclusive anisotropic model, can be employed not only to cope with azimuthal velocity variation, but also to obtain significant detail of fracture networks (Figure 2c).

Orthorhombic anisotropy may be the simplest realistic symmetry for many geophysical problems (Tsvankin, 1997). One of the most common reasons for orthorhombic anisotropy in sedimentary basins is a combination of parallel vertical cracks and vertical transverse isotropy in the background medium (Wild and Crampin, 1991). Orthorhombic symmetry can also be caused by two or three mutually orthogonal crack systems or two identical systems of cracks making an arbitrary angle with each other. The theory and algorithm for imaging in orthorhombic velocity media was developed by Tsvankin (1997) for weak anisotropy and Xie et al. (2011) for stronger anisotropy. An orthorhombic system is defined by the three mutually orthogonal symmetry planes.

(a) (b) (c)

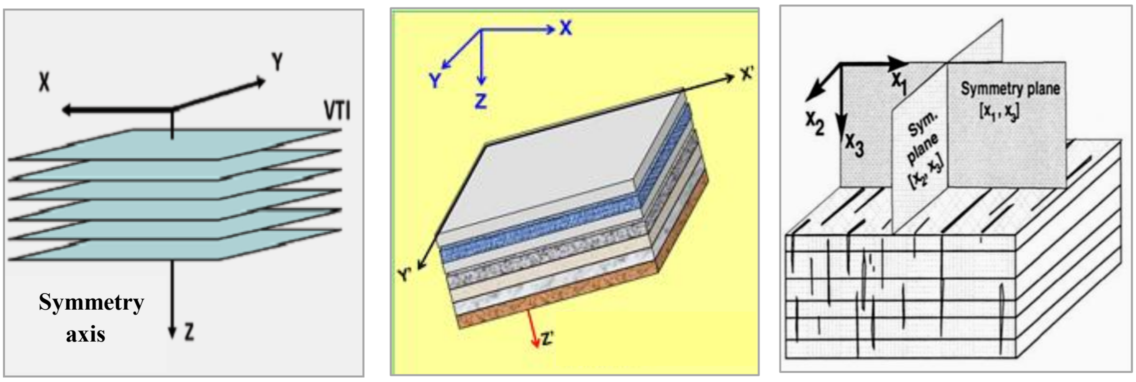

Figure 2. Anisotropic models: (a) VTI, (b) TTI and (c) orthorhombic (Tsvankin, 2001).

The seismic wave equation operates as a kernel of imaging and inversion algorithms. To appreciate the influences of seismic anisotropy in imaging, the isotropic wave equation must be replaced by the anisotropic one. Although seismic anisotropy is inherently an elastic phenomenon, the elastic anisotropic wave equation is rarely used in anisotropic imaging techniques due to its heavy computational process. Pseudo-acoustic approximations have been suggested to mitigate the computational cost (Fletcher *et al.*, 2008).

Different approximations, such as weak anisotropy (Thomsen, 1986), elliptical (Dellinger and Muir, 1988), and anelliptic approximations (Dellinger *et al.*, 1993; Fomel, 2004; Alkhalifah and Tsvankin, 1995), are proposed to simplify the VTI anisotropic equation. In reality, it is rare to have a media with elliptical or weak anisotropy properties, however, anellipticity (deviation of wavefield from ellipse) have been commonly observed in the Earth subsurface, and it is a significant characteristic of elastic wave propagation (Stovas and Fomel, 2012; Fernandes *et al.*, 2015).

Alkhalifah (1998) first demonstrated that by setting the vertical S wave velocity to zero for VTI media, one can extract much a simpler dispersion relation than the elastic expression. He thereafter developed an acoustic VTI wave equation by using the dispersion relation which yielded acceptable approximations to the elastic equation (Alkhalifah, 2000). Several pseudo-acoustic wave equations were later derived based on Alkhalifah's dispersion relation (Du *et al.*, 2007; Hestholm, 2007).

Another approach, to obtain a pseudo-acoustic wave equation, uses Hooke's law and the equations of motion in which the vertical S wave velocity is also considered equal to zero (Duveneck *et al.*, 2008; Zhang and Zhang, 2009). In both methods, by taking the tilt of the symmetry axis into account, pseudo-acoustic TTI approximations can be achieved (Fletcher *et al.*, 2009). Although the pseudo-acoustic approximation performs well in isotropic and elliptical anisotropic ($\varepsilon = \delta$, ε and δ are Thomsen's parameters) conditions, for anellipticity, where $\varepsilon \neq \delta$, the S wave velocity is only zero along the symmetry axis. In other directions, the S wave velocity has a value and when the approximation is applied for imaging, Sv wave components add noise into the images (Figure 3) (Grechka *et al.*, 2004).

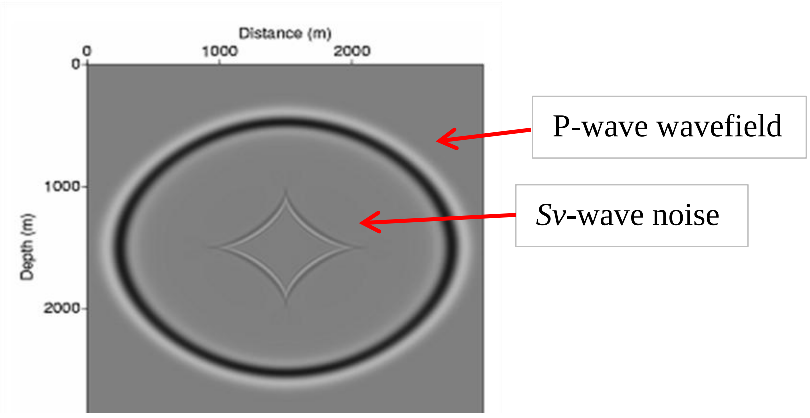

Figure 3. Pseudo-acoustic wavefield with shear wave noise at the center (Fletcher *et al.*, 2008).

Various methods have been suggested to solve the problems involved in the pseudo-acoustic approximations. One way to remove S-wave artifacts is to locate the source in either an isotropic or elliptical anisotropic environment (Alkhalifah, 2003). However, P-waves can still convert to Sv waves due to interfaces with sharp acoustic impedance contrast while waves propagate in the subsurface. One can eliminate the converted S waves either by adding finite Sv wave velocities along the axis of symmetry (which also fixes the instability problem (Fletcher *et al.*, 2009)), or by implementing a filter at each output time step (Zhang *et al.*, 2009). An efficient approach to remove the Sv wave mode completely is to derive pure acoustic wave equations.

Several pure P-wave approximations have been developed for VTI, TTI and orthorhombic media. Klíe and Toro (2001) modified Alkhalifah's acoustic wave equation (Alkhalifah, 2000). Their proposed equation does not create any S-wave artifacts, and is applicable for any anisotropic strength. Du et al. (2007) derived P- and S_V-wave equations for TTI media based on the weak anisotropy approximation to apply in a reverse time migration. The equations were written in the time-wavenumber domain, and the pseudo-spectral method was used for numerical solutions.

Liu et al. (2009) and Du et al. (2010) further developed new VTI pure P-wave propagators from the optimized separable approximation. Decoupling the P-S_V dispersion relation and deriving separate equations for P- and S_V-waves resulted in unconditionally stable and free S-wave solutions for P-wave equations (Figure 4). Given that shear and mode-converted waves carry valuable information about the anisotropy of the Earth, in these methods, shear wave equations can be implemented for anisotropic S-wave modeling (Cheng and Kang, 2014).

Chu et al. (2012) developed a pure acoustic VTI wave equation, which can be conveniently solved using the pseudo-spectral approach. They started from the pure P-wave dispersion relation proposed by Chu et al. (2011) and showed that it could be rewritten in a summation form. The pseudo-analytical method is another way to approximate the pseudo-differential operator using interpolation of several constant parameter modeling responses, similar to the phase shift plus interpolation algorithm for one-way wave propagation (Gazdag and Sguazzero, 1984; Etgen and Brandsberg-Dahl, 2009; Crawley *et al.*, 2010).

The lowrank approximation, the rapid expansion and the Fourier finite difference (FFD) methods are improved versions of the pseudo-analytical technique that was introduced by Fomel et al. (2010), Pestana and Stoffa (2010), and Song and Fomel (2011). Moreover, Kang and Cheng (2011) introduced new coupled second-order systems called pseudo-pure-mode wave equations (Cheng and Kang, 2012). They separated P-wave and S-wave data by applying a filtering step to correct projection deviations resulting from the variation between polarization direction and its isotropic references (Cheng and Kang, 2014).

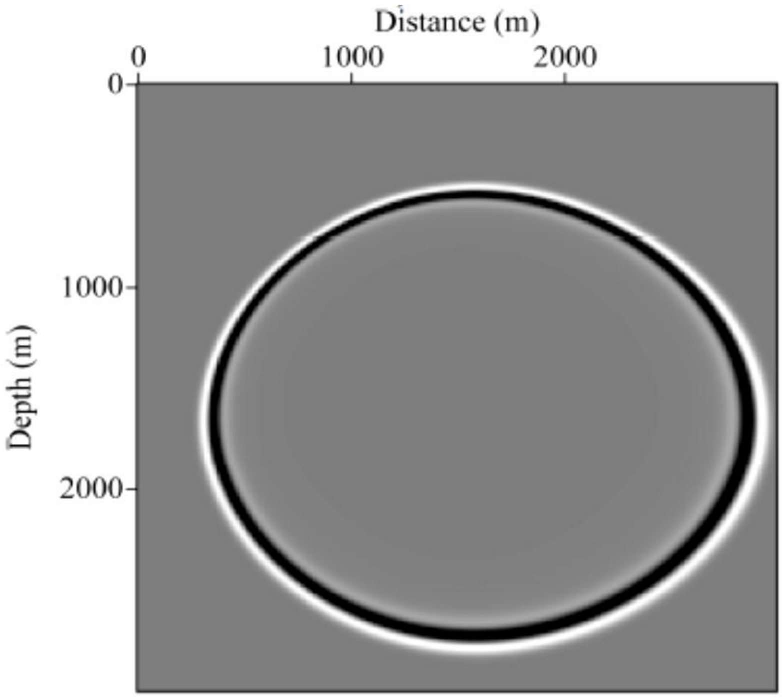

Figure 4. Impulse response of a VTI pure P-wave propagator (Du et al., 2010).

Besides wave modeling and creating synthetic seismic data, the above-mentioned methods can be employed for imaging purposes. For instance, by applying a wave equation into the eikonal equation, one can compute the traveltime of a wave from source to receiver which is needed for Kirchhoff depth imaging. Moreover, the wave equation is directly used in RTM technique. In the next section, more detail about Kirchhoff imaging is reviewed.

3. Anisotropic Kirchhoff Depth Imaging

In many parts of the world, prestack time migration (PSTM) is applied as a common approach of seismic imaging in the industry. The reason for this is the simple efficiency and robustness of time imaging and its ability to focus seismic reflectors for many geological settings. Another reason for the continued widespread use of time imaging is that interpreters have traditionally been used to looking at images in the time domain and often prefer to continue to do so. Yet, limitations of PSTM appear in the case of strong lateral velocity variations, where the more rigorous imaging and more accurate velocity models, offered by prestack depth migration (PSDM), are required (Guo and Fagin, 2002; Lambaré *et al.*, 2007).

Figure 5 illustrates the difference of PSTM and PSDM to image the subsurface events. It can be clearly seen that PSTM has failed to detect the reflectors (top), and to locate them in the right place (bottom)(Guo and Fagin, 2002). Another example is illustrated in Figure 6. It is explicit that the time image shows the top basement reflector (indicated by yellow arrows) closed and continuous. However, the depth image provides more detail about the structure, and the reflector is detached in few places due to tectonic activities.

(a)

(b)

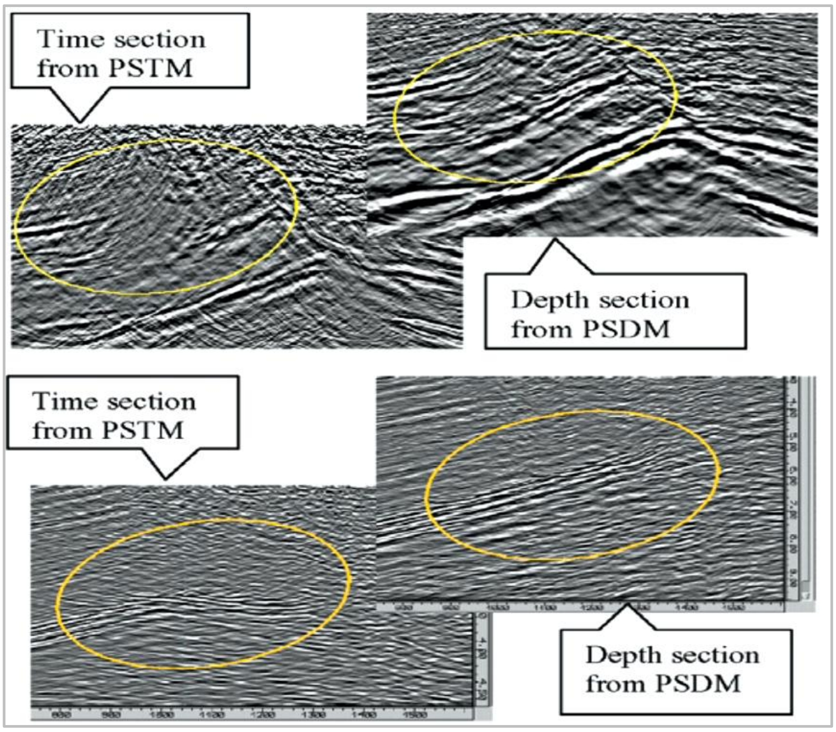

Figure 5. Comparison of PSTM and PSDM: (a) an imaging problem, (b) a positioning problem (Guo and Fagin, 2002).

Computing capacity used to be a key limitation in the use of depth imaging, but with today's modern computer clusters this is no longer a major issue. In parallel to the increase in computing capacity, a large range of depth imaging methods have been developed and are now used in the seismic industry (Vinje, 2010). PSDM has become the most widely used imaging technique in seismic exploration because of its high accuracy for complex subsurface structures such as salt domes, thrust belts, faults, and stratigraphic structures (Jang and Kim, 2011; Brown *et al.*, 2013).

(a)

(b)

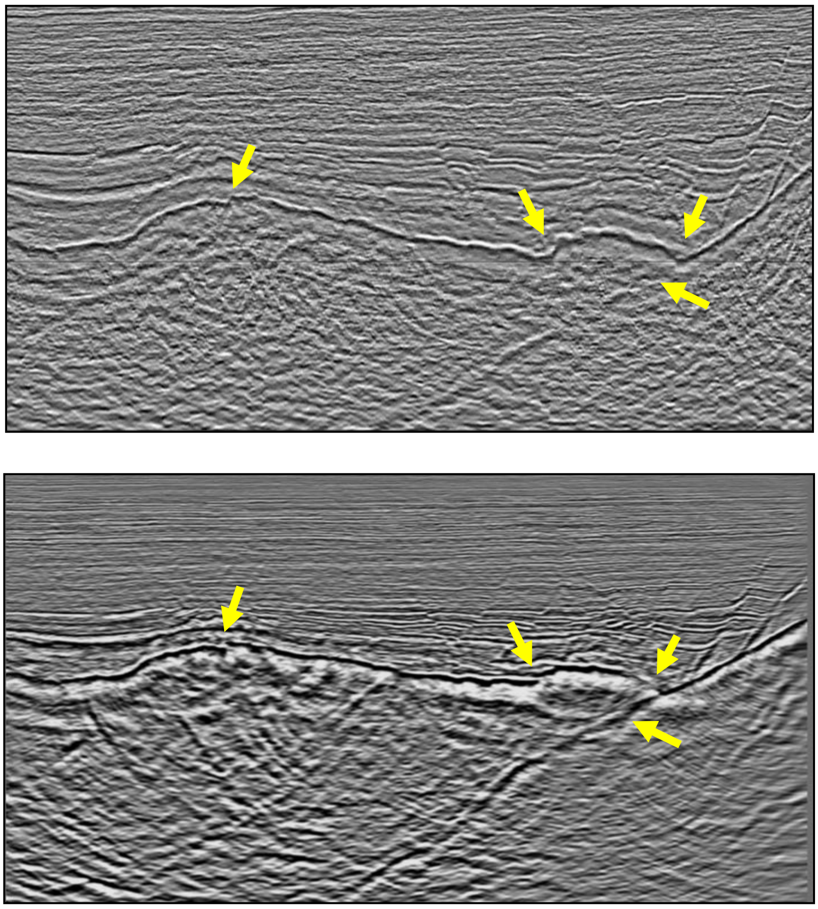

Figure 6. Exploration of fractures igneous basement using (a) Kirchhoff time migration, and (b) Kirchhoff depth migration (courtesy of PETRONAS Vietnam).

In depth imaging, prestack migration avoids numerous assumptions for stacking the data which definitely restrict the effectiveness of imaging (Whiting *et al.*, 2003). Stacking the data leads to lose valuable velocity analysis information. Imaging pre-stack data can take all traces for migration before being moved to zero-offset. Figure 7 indicates the differences between a poststack and a prestack image of a dataset. It is obvious that the resolution and focusing of interfaces in the prestack image is higher than the poststack image (Bancroft, 2007).

However, one issue in PSDM is using isotropic approximation which still can introduce errors in focusing of the seismic energy. The seismic images produced in depth by isotropic approaches rarely tie the formation depths that are observed in well bores. Accurate imaging in depth requires an anisotropic representation of earth parameters (Bowling *et al.*, 2009). Figure 8 compares the image of an isotropic PSDM with an anisotropic PSDM. The positioning of interfaces and the resolution of image are corrected in anisotropic approach (Epili *et al.*, 2011).

Over the years, several types of depth imaging methods have been developed. They can be classified by the way they propagate the wavefield in the model. A numerical solution of the acoustic wave equation is used in RTM and one-way wave equation migration, while a solution of the eikonal equation using a ray approximation is used in Kirchhoff and beam migration (Griese, 2010). Although RTM and the beam migration method provides higher quality images than Kirchhoff migration (Schneider, 1978), this classical migration method is still one of the most popular seismic imaging methods due to its great efficiency and flexibility for time and depth imaging (Etgen *et al.*, 2009). This method is suitable to solve imaging problems in areas with smooth velocity variations but also leads to accurate results in areas with moderately complex geologic structures.

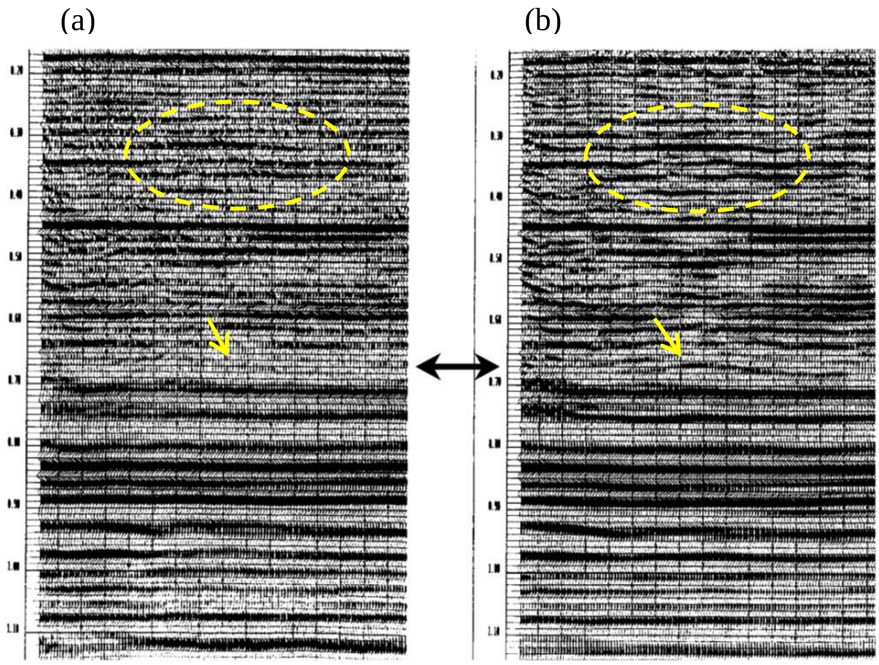

Figure 7. A comparison of (a) a poststack, and (b) a prestack migration (Bancroft, 2007).

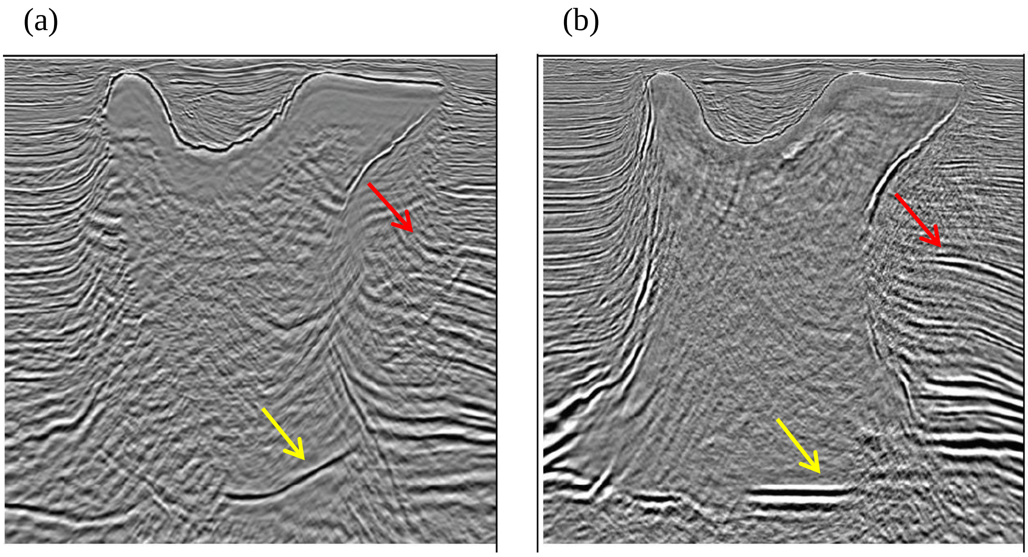

Figure 8. Depth imaging with (a) isotropic parameters, and (b) anisotropic (TTI) parameters (Epili *et al.*, 2011).

A standard workflow for Kirchhoff depth imaging is presented in Figure 9. Figure 10 compares the cost and the image quality of different imaging techniques such as Kirchhoff PSTM, Kirchhoff PSDM, fast beams, full beam PSDM and RTM. PSTM and fast beams are very fast, but their efficiency is low. Kirchhoff PSDM creates better images with slightly more cost. RTM and full beam PSDM produce excellent results yet they are very expensive.

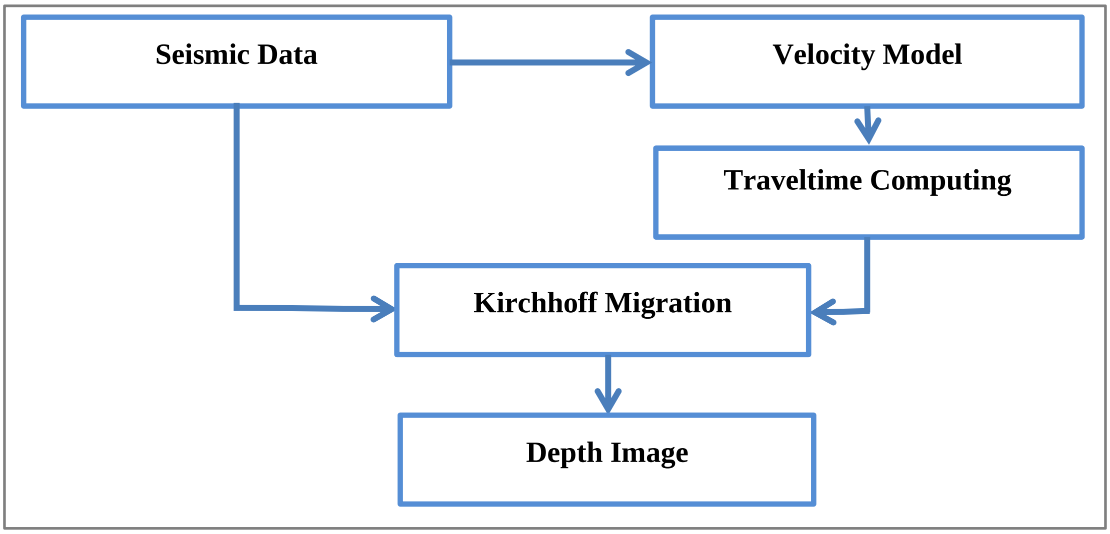

Figure 9. A standard workflow for Kirchhoff PSDM.

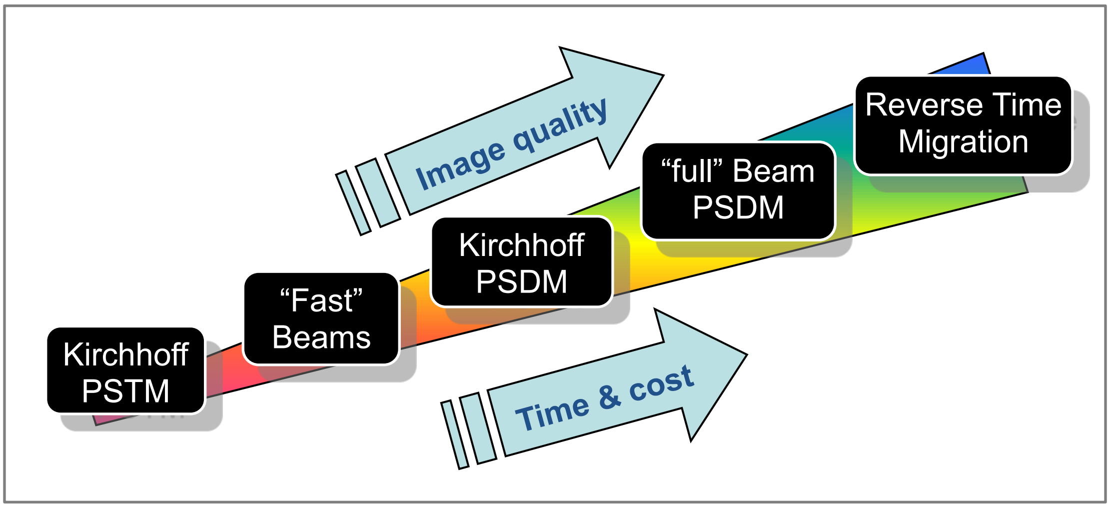

Figure 10. Schematic comparison of seismic imaging methods in terms of their cost and image quality.

In Kirchhoff PSDM, a challenging issue is the computation of traveltime taken by seismic wave from source to receiver. In P-wave velocity analysis and imaging, several traveltime computing algorithms have been developed for VTI condition (Behera *et al.*, 2011; Koren *et al.*, 2008; Hao *et al.*, 2011; Zhu *et al.*, 2005). There are two main classes of traveltime computing which comprise ray tracing and eikonal solver methods (Alkhalifah and Fomel, 2010; Ai-Hua *et al.*, 2006). Ray tracing utilizes the eikonal equation and a system of ordinary differential equations to compute traveltimes along the rays . The ray technique was firstly employed to study the propagation of the high-frequency elastic waves by Babich (1956) and by Karal and Keller (1959). Several studies have been conducted by applying ray tracing technique in various anisotropic models (Cerveny, 2005; Wang, 2014).

The main algorithms in ray tracing are defined as shooting and bending methods. The shooting approach utilizes a given initial direction and position for the ray and an interpolation tool to reach a certain point which is also known as initial-value method (Figure 11). For modeling and imaging of complex structures by using shooting method, one can compute the multiple arrivals containing the most energetic wave which does not necessarily correlate to the first arrival (Cerveny, 2005).

Sun and Schuster (Sun and Schuster, 2003) employed the initial-value approach for the wave path migration. Julian (Julian, 1970) developed an initial-value method to track rays in inhomogeneous media which is applied by Engdahl (Engdahl, 1973) for earthquake analysis. Since an interpolation between rays is needed to calculate traveltimes, it causes difficulty for this technique specifically in complex structures where a divergence may happen to the rays (Waheed *et al.*, 2015).

An efficient alternative to compute traveltimes is solving the eikonal equation by employing finite differences (Waheed *et al.*, 2015; Sethian and Popovici, 1999). Although eikonal solvers only provide the first arrival times, they can compute traveltimes between two points. An attempt to extend eikonal solvers to obtain multiple arrivals was conducted by Bevc (1997). Different techniques have been introduced to solve the eikonal equation, such as embedding methods, single-pass methods, sweeping methods, and iterative methods (Waheed *et al.*, 2015).

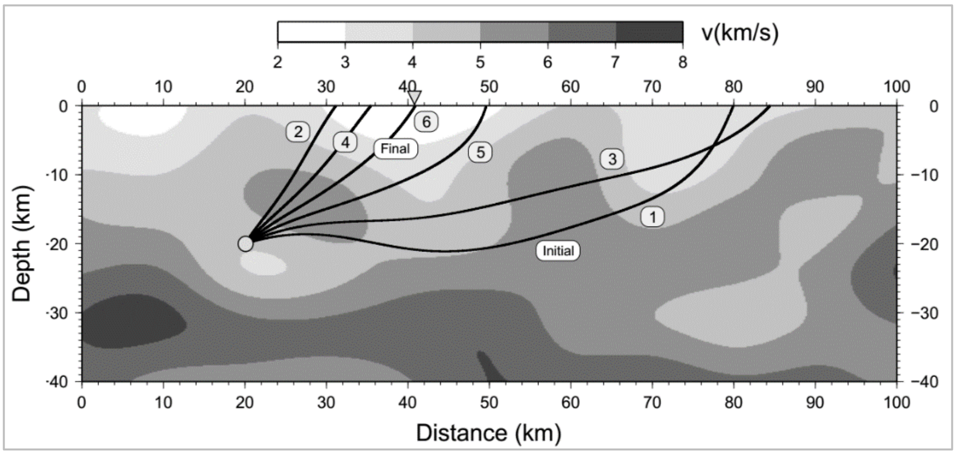

Figure 11. Principle of the shooting (initial-value) technique. The initial path trajectory is updated until it converges at the receiver (Rawlinson *et al.*, 2008).

The main difference of these techniques is in how they cope with the complication of multi-valued solutions and in finding solutions in the vicinity of cusps and discontinuities (Sava and Fomel, 1998). Anisotropy is initially added to an eikonal solver algorithm by Dellinger (1991). The embedding and iterative methods are both time consuming, particularly in heterogeneous and anisotropic conditions (Waheed *et al.*, 2015). Fast sweeping methods are originally proposed for isotropic media (Zhao, 2004), however, a modification is executed to handle the anisotropic condition (Zhang *et al.*, 2006).

Single-pass or fast marching method (FMM) is another tool for computing traveltimes but is not generally applicable for anisotropic medium. Sethian and Popovici (1999) demonstrated a very fast, accurate and stable traveltime computation algorithm based on solving the isotropic eikonal equation using FMM. The finite difference approximation to the eikonal equation is resolved in each point of the grid, making the accuracy of the method dependent only on the grid size and the order of the finite difference scheme used.

The stability and speed of the method are ensured by following the wavefront propagation in a narrow band and solving the finite difference eikonal stencil at points of minimum traveltime in the narrow band. They mentioned that the FMM can be used in many geophysical applications requiring modeling wave propagation in complex geological media: 3D Kirchhoff prestack and poststack migration, 3D datuming, 3D velocity analysis, 3D Kirchhoff modeling, and 3D controlled illumination modeling (Moussavi Alashloo *et al.*, 2016). The FMM allows for flexible implementations in either Cartesian or spherical coordinates.

This algorithm has since been modified to work for anisotropy (Sethian and Vladimirsky, 2001; Cristiani, 2009). Waheed and Alkhalifah (2013) employed the FMM on the VTI eikonal equation approximated by Alkhalifah (1998) to compute traveltimes. They used perturbation theory to solve the quartic traveltime polynomial resulting from the discretized form of the VTI eikonal equation. The computational cost of the algorithm is 16 percent of the exact VTI solver which shows its efficiency. The suggested approach relies on the application of perturbation theory to the discretized VTI eikonal equation. The accuracy of the method is compared with the exact VTI solver in Figure 12. Although computing is fast and stable, the accuracy need to be improved. One of the key parameters for computing the traveltime is the wave equation utilized in eikonal equation. As it is discussed in previous section, many approximations have been derived for wave propagation. Among these numerous assumptions, Fomel (2004) proposed an extension of the Muir-Dellinger approximations (Dellinger *et al.*, 1993) using the shifted-hyperbola approach. The resultant three-parameter approximation for phase velocity was identical to the acoustic approximation proposed by Alkhalifah (2000).

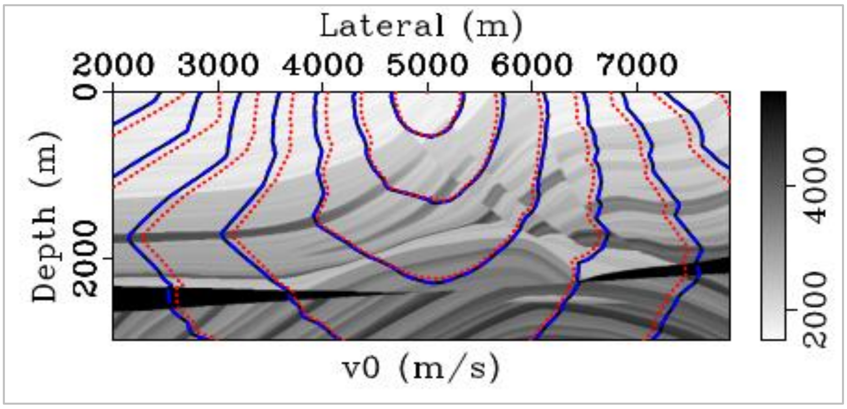

Figure 12. Traveltime contours for VTI Marmousi model using the exact VTI solution (dashed blue), and using the application of perturbation theory on the VTI acoustic approximation (dotted red) mapped on the velocity model. The source is located at x = 5000 m on the surface (Waheed and Alkhalifah, 2013).

Moussavi Alashloo and Ghosh (2018) utilized a fast marching eikonal solver in the isotropic and VTI concepts. They also tested the results by using the Kirchhoff depth migration algorithm. The comparison of isotropic and VTI traveltimes demonstrates a considerable lateral difference among wavefronts (Figure 13). The results of Kirchhoff imaging show that the VTI algorithm generates images with perfect positioning and higher resolution than the isotropic one, specifically in deep areas (Figure 14).

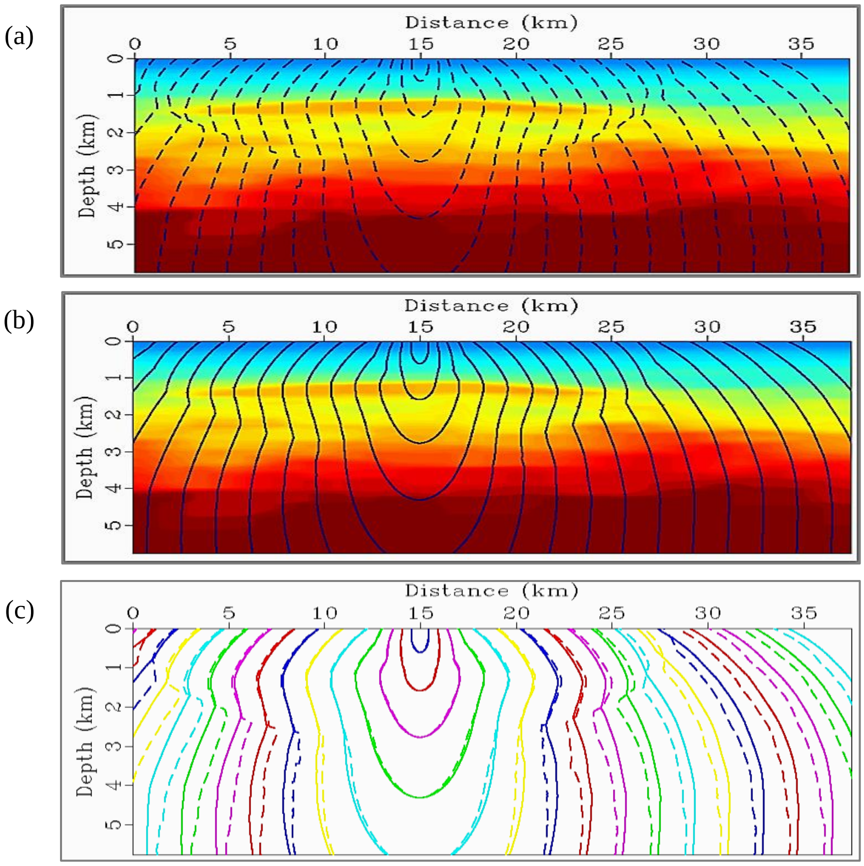

Figure 13. (a) Isotropic traveltime contours, and (b) anisotropic traveltime contours for a source at x=15 km. (c) Comparison of isotropic traveltimes with dashed curves and VTI traveltimes with solid curves.

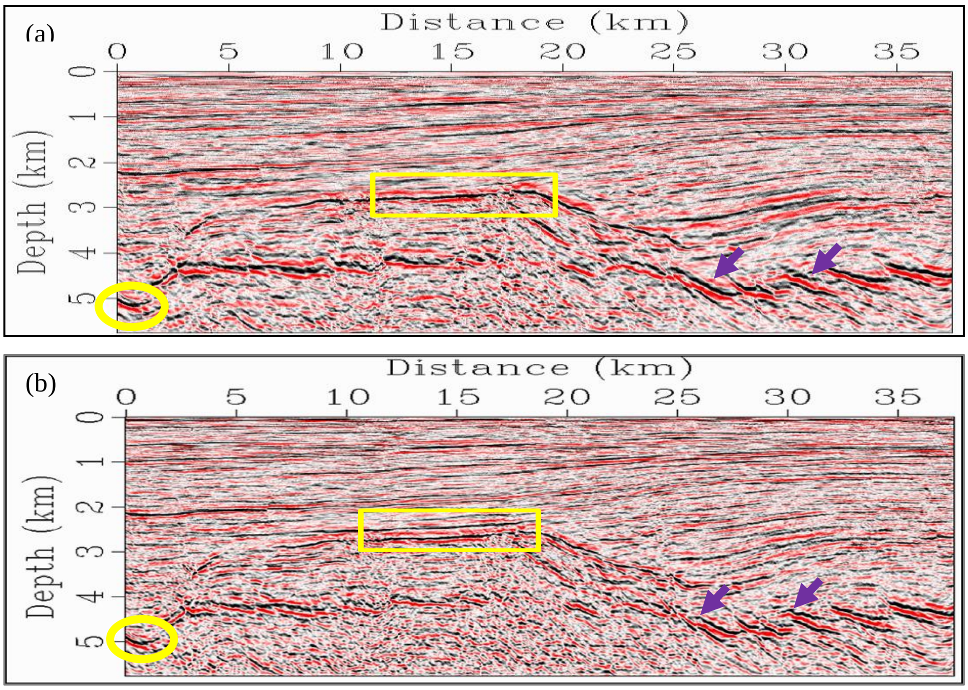

Figure 14. Comparison between (a) isotropic and (b) anisotropic images.

4. Conclusion

One of the features of anisotropy is that, however generally it is weak in many contexts, it affects seismic data remarkabely. Since most of hydrocarbon reservoirs are defined as anisotropic media, considering anisotropy is necessary not only to avoid distortions in imaging, but also provides valuable information about lithology and fracture networks. To consider the influences of seismic anisotropy, an anisotropic wave equation needs to be employed. Depending on the type of anisotropic model, various wave equations are introduced, which can be used for both seismic modeling and imaging. Several approximations are proposed, such as weak anisotropy, and linear approximations, however, they are too simple to cover all aspects of anisotropy. Non-linear assumption is a more realistic approximation, which is applicable whether for weak and strong anisotropy, or equal and unequal Thomsen's parameters. It also has been proven that isotropic analysis is not able to generate flat events in seismic imaging, and hockey sticks still exist, while anisotropic analysis provides flat events. Therefore, in high-resolution seismic methods, anisotropy must be considered to provide the details required for production delineation of reservoir, details so critical to the production engineers.

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